From Calculus to Cohomology

Homework 2

Exercise 1. Prove the lemma from the lecture saying that $\omega \in Alt^k(V)$ if and only if $\omega(v_1, \dots, v_k) = 0$ whenever $v_i = v_{i+1}$, for some $i = 1, \dots, k-1$.

Exercise 2. Prove that for $\omega_1 \in Alt^k(V)$ and $\omega_2 \in Alt^m(V)$

$$\omega_1 \wedge \omega_2 = (-1)^{km} \omega_2 \wedge \omega_1.$$

Exercise 3. Prove that for $\omega_1 \in Alt^k(V)$, $\omega_2 \in Alt^l(V)$, and $\omega_3 \in Alt^m(V)$

$$(\omega_1 \wedge \omega_2) \wedge \omega_3 = \omega_1 \wedge (\omega_2 \wedge \omega_3).$$

Exercise 4. Find $\omega \in Alt^2(\mathbb{R}^4)$ with $\omega \wedge \omega \neq 0$.