

# From Calculus to Cohomology

## Homework 1

**Exercise 1.** Using the definitions of *grad*, *rot*, *div* given in the lecture for the case of subsets of  $\mathbb{R}^3$ , show that

$$\text{rot} \circ \text{grad} = 0, \quad \text{div} \circ \text{rot} = 0.$$

**Exercise 2.** Prove that if  $U \subset \mathbb{R}^3$  is star shaped then  $H^1(U) = 0 = H^2(U)$ . What is  $H^0(U)$  then?  
*Hint:* You may want to play a bit with integrating and with a function  $(x_1, x_2, x_3) \mapsto (f_2x_3 - f_3x_2, f_3x_1 - f_1x_3, f_1x_2 - f_2x_1)$  evaluated at  $tx$ .

**Exercise 3.** Decide whether  $H^1(\mathbb{R}^3 \setminus S)$  is  $\{0\}$  or not, if  $S = \{(x_1, x_2, 0); x_1^2 + x_2^2 = 1\}$ .

*Hint:* You may want to play a bit with a function  $(x_1, x_2, x_3) \mapsto \frac{1}{x_3^2 + (x_1^2 + x_2^2 - 1)^2} (-2x_1x_3, -2x_2x_3, x_1^2 + x_2^2 - 1)$ .