Systematic Beam Management in mmWave Network: Tradeoff Among User Mobility, Link Outage, and Interference Control

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***Abstract*—In this paper, we study the beam management in mmWave network from a systematic perspective. First, instead of optimizing the network performance regarding the sum-rate, we improve the beam coverage to better support user mobility, which can further reduce the beam tracking overhead. Second, to compensate for the severe signal attenuation in mmWave band, we configure the beam so as to satisfy the user link outage probability constraint. Third, to reduce the interference among users, we consider the inter-beam interference from both main-lobe and side-lobe. We formulate our beam management scheme as a non-linear integer optimization problem, which typically has high computational complexity. By deliberately transforming it to a geometric optimization problem and designing the rounding method, we give an optimized feasible solution with low computational complexity. We verify the performance of our beam management scheme through simulation. Extensive simulation results demonstrate that our proposed algorithm can efficiently manage the beam in mmWave network.**

***Index Terms*—mmWave, beam management, user mobility, interference control, link outage.**

I. Introduction

The exponential increase of mobile data imposes great challenges to the current wireless network. However, due to the spectrum scarcity in sub-6GHz band, further improving the network performance in this band becomes more and more challenging. Therefore, expanding the network operating frequency to mmWave band is a foreseen approach. Typically ranging from 30GHz to 300GHz, the mmWave frequency has a wide bandwidth, thus can significantly improve the network performance in the perspective of data rate. Unfortunately, the mmWave band faces severe signal attenuation compared with the sub-6GHz band [1]. As a consequence, the massive array antenna has to be exploited to compensate for the severe signal attenuation, under the cost of introducing the beamforming and beam management problem. There have been quite a lot of literatures in this area in the recent years.

For the beamforming in mmWave band, since the array elements number is much larger than that of sub-6GHz, to reduce the hardware cost, the traditional digital beamforming method [2] is not applicable. Therefore, the analog beamforming [3] and digital-analog hybrid beamforming [4]–[8] techniques attract more and more attentions.

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Applying the beamforming to mmWave network, a key question is how to match users and beams so as to improve the network performance. For the digital-analog hybrid beamforming, a magnitude maximization scheme was designed in [9]. To further improve its performance, several beam selection schemes were studied separately in [10] and [11]. Besides, greedy and maximum weight matching schemes were proposed in [12]. These beam selection approaches require the channel state information (CSI) to make the decision. However, such CSI can hardly obtain due to the large control overhead. While for the analog beamforming, the low-complexity beam selection method was proposed in [13], and the RF chain constraint was further considered in [14]. To optimize the worst-case user performance, an adaptive frequency-reuse method was studied in [15] to decrease the inter-beam interference among users.

Different to the above work, we study the beam management of mmWave network in a systematic insight. Specifically, from the perspective of increasing the link budget, it is preferred to have a larger beamforming gain so as to reduce the link outage probability. However, increasing the beamforming gain typically brings a narrower main-lobe beamwidth, thus cannnot well support user mobility due to the limited beam coverage. As a consequence, the beam tracking has to be frequently conducted to track the mobile user, which can introduce high network control overhead [16]. Moreover, the inter-beam interference, which may come from both main-lobe and side-lobe, has also to be carefully considered to reduce the mutual interference among users. Therefore, for the beam management in mmWave network, it is of paramount importance to jointly study the user mobility, link outage, and interference control.

Motivated by the above, in this paper, we systematically look into the beam management of mmWave network. We study the problem in dimensions of user mobility, link outage, and interference control. The main contributions of this paper are as follows:

1) Instead of optimizing the network performance regarding the widely adopted sum-rate criterion, we manage the beam to optimize its coverage to better support user mobility;
2) To compensate for the severe signal attenuation in mmWave band, we study the beam management to

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satisfy the link outage constraint;

3) To reduce the interference among users, we consider the interference from both main-lobe and side-lobe by carefully designing the main-lobe and side-lobe pattern;
4) We jointly formulate our beam management problem as a non-linear integer optimization problem, which typically has high computational complexity. By deliberately transforming it to a geometric optimization problem and designing the rounding method, we can efficiently derive an optimized feasible solution, which has low computational complexity.

II. mmWave Network Model

We consider an mmWave network, in which the AP is deployed with a massive uniform line array (ULA) antenna with the sum element number N. The user is deployed with the omni-directional antenna. We consider an analog beamforming ULA antenna architecture, where the antenna element spacing is no larger than the half wave length, so as to avoid the grating lobes. We assume that the radiation power of each antenna element is a constant, and is denoted as p_t. To serve multiple users, as shown in Fig. 1, we suppose that the antenna array can be configured as different sub-arrays to form multiple beams. The number of antenna element in each sub-array is optimized to form a preferred radiation pattern. The maximum number of simultaneously serving user is K, which is constrained by the number of RF chain. We assume $N \gg K$. Once the sub-array configuration has been determined, the traditional beamforming method [17] can be adopted to form a directional beam for the serving user.

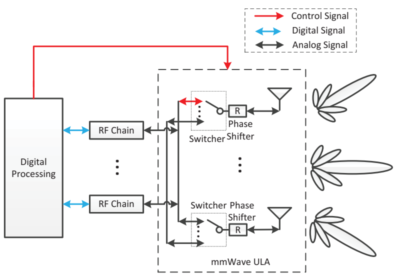

Fig. 1. mmWave Array Architecture

In the network association process, since the AP has no information about users in either steering direction or the signal attenuation, the AP explores all its antenna elements to form a high-gain sweeping beam to the user. After receiving the association frame, the user can know the AP beam index and measure the received signal power. Feeding these information back by the associated users, the AP can know the steering direction through the beam index, and the average path loss from the RSSI (Received Signal Strength Indicator). Using these information, the AP can configure its array antenna to simultaneously service multiple users. We denote i as the user index, and n_i as the element number allocated for serving user i. Due to the severe signal attenuation in mmWave band, we only focus on the line of sight (LOS) channel as in [9].

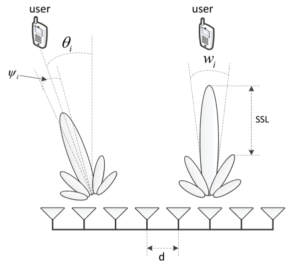

Fig. 2. ULA Model

A. Property of the ULA

As in Fig. 2, we denote $\theta_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$ as the steering direction for user i, d as the element spacing, and λ to be the wave length. For the ULA antenna with element number of n_i, the antenna gain g_i for the user i is proportional to n_i, and can be written as

$$g_i = \frac{4\pi d^2 n_i \cos\theta_i}{\lambda^2}.$$

The side-lobe level (SLL), defined as the amplitude at the peak of the main-lobe to the maximum peak among side-lobes, can be 12dB if the element number $n_i \geq 5$ [17].

For the first null beamwidth, it is inversely proportional to the element number n_i, and can be written as

$$w_i = \frac{2\lambda}{dn_i\cos\theta_i}\text{rad} = \frac{2\lambda}{dn_i\cos\theta_i}\frac{180}{\pi}\text{deg}.$$

While for the 3dB beamwidth, it can be derived as

$$\psi_i = \frac{2.78\lambda}{d\pi n_i\cos\theta_i}\text{rad} = \frac{2.78\lambda}{d\pi n_i\cos\theta_i}\frac{180}{\pi}\text{deg}.$$

B. Main-lobe Interference Constraint

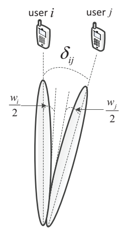

Fig. 3. Interference From the Main-lobe

To alleviate the inter-beam interference among the simultaneously serving users, as showed in Fig. 3, the first null beam width serving for user i and j should be non-overlapped. This implies that

$$\frac{w_i}{2} + \frac{w_j}{2} \leq \delta_{ij},$$

where δ_{ij} is the angle between user i and j.

C. Side-lobe Interference Constraint

The side-lobe can also introduce interference among users. We consider the worst-case interference. Without loss of generality, we assume that the main-lobe gain of user i is smaller than that of user j as in Fig. 4. We discuss the side-lobe interference as follows.

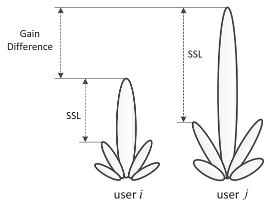

Fig. 4. Interference From the Side-lode

1) Side-lobe Interference from User i to User j: Since the radiation power of each antenna element is a constant, compared with the received signal power of user j, the interference power from the side-lobe of user i is smaller than the SSL of user i. Therefore, bounding the SLL for the configured beam of user i could reduce its side-lobe interference. From subsection II-A, we can derive that if $n_i \geq l$, where l is a pre-determined parameter, the SLL constraint can be fulfilled. Then, to reduce the side-lobe interference from user i to user j, we have $n_i \geq l$.

2) Side-lobe Interference from User j to User i: For the interference to user i from side-lobe of user j, besides the SLL constraint for the beam of user j, we further have to put a constraint on the difference of main-lobe gain between user i and j. Therefore, we have the following constraints

$$n_j \geq l \text{ and } \max_{i,j} \left\{ \frac{n_j \cos\theta_j}{n_i \cos\theta_i} \right\} \leq \xi,$$

where l and ξ are pre-determined parameters. As a result, compared with the received power of user i, the side-lobe interference from user j is smaller than $\mathrm{SLL}_j(\mathrm{dB}) - \xi(\mathrm{dB})$.

D. Link Outage Probability Constraint

The attenuation between AP and user i consists of two parts: the path loss L_i and channel fading Γ_i. As stated in the subsection II-A, using the beamforming at the AP side, the antenna gain g_i can be obtained for user i. Therefore, the received power p_i at user i can be written as

$$p_i(\mathrm{dBm}) = p_t(\mathrm{dBm}) + g_i(\mathrm{dBi}) - L_i(\mathrm{dB}) - \Gamma_i(\mathrm{dB}).$$

Using the interference control method in Subsection II-B and II-C, the interference power can be much smaller than the received power (e.g. 10 dB), and can be ignored. Hence, denoting the receiver sensitivity as α, the link outage probability [18] c_i of user i can be written as

$$\Pr\{p_i \leq \alpha\} \leq c_i,$$

where c_i is a pre-defined parameter.

Under the assumption of LOS channel, we have that the received power follows the non-central χ-square distribution. Given α and c_i, the above link outage probability constraint can be transformed to the following

$$g_i = \frac{4\pi d^2 n_i \cos\theta_i}{\lambda^2} \geq h(c_i, \alpha, \Gamma_i, L_i, p_t),$$

where $h(c_i, \alpha, \Gamma_i, L_i, p_t)$ is a numerically derived constant [19] regarding c_i, α, Γ_i, L_i, and p_t.

E. Array Element Number Constraint

Since the sum element number is N, the array element constraint can be described as $\sum_i n_i \leq N$.

F. Beamwidth Coverage Optimization

To support the user mobility, it is preferred to configure a larger 3dB beamwidth. Further, to maintain fairness among users, we choose the logarithm function, which has a diminishing marginal utility, as our objective function. Hence, we have the following beamwidth optimizing objective $\sum_i \log\left(\frac{2.78\lambda}{d\pi n_i \cos\theta_i} \frac{180}{\pi}\right)$.

G. Beam Management Problem

Based on the above analysis, we can write our beam management problem in mmWave network as follows:

$$\max_{n_i} \sum_i \log\left(\frac{2.78\lambda}{d\pi n_i \cos\theta_i} \frac{180}{\pi}\right) \quad (1)$$

$$\text{s. t. } \frac{4\pi d^2 n_i \cos\theta_i}{\lambda^2} \geq h(c_i, \alpha, \Gamma_i, L_i, p_t), \forall i, \quad (2)$$

$$\frac{\lambda}{d n_i \cos\theta_i \delta_{ij}} + \frac{\lambda}{d n_j \cos\theta_j \delta_{ij}} \leq 1, \forall i, j, i \neq j, \quad (3)$$

$$\max_{i,j} \left\{ \frac{n_i \cos\theta_i}{n_j \cos\theta_j} \right\} \leq \xi, \forall i, j, i \neq j, \quad (4)$$

$$n_i \geq l, \forall i, \quad (5)$$

$$\sum_i n_i \leq N, \quad (6)$$

$$n_i \in \mathbb{N}, \forall i. \quad (7)$$

For the above optimization problem, the objective (1) is to maximize the 3dB beamwidth to better support the user mobility. Constraint (2) is to guarantee the outage probability, constraint (3) is to ensure non-overlapping among the main-lobes so as to control the interference from main-lobe, constraint (4) and (5) are to control the interference from the side-lobe, constraint (6) is the array element number constraint, and (7) is the integer constraint.

III. Beam Management Strategy

The above optimization problem (1)-(7) is a non-linear integer optimization problem, and has high computational complexity. Since the beam has to be configured online, it has to be efficiently solved, which is discussed in this section.

A. Problem Relaxation

To solve the optimization problem (1)-(7), we have to derive an optimized feasible solution. To achieve this, we first refine the array element number constraint in (6) from N to $N-K$. Then, we replace the side-lobe constraint (4) by

$$\max_{i,j} \left\{ \frac{(n_i+1)\cos\theta_i}{n_j \cos\theta_j} \right\} \leq \xi, \forall i, j.$$

Last, we relax the integer constraint (7). Thus, we have the following optimization problem

$$\max_{n_i} \sum_i \log\left(\frac{2.78\lambda}{d\pi n_i \cos\theta_i}\frac{180}{\pi}\right) \tag{8}$$

$$\text{s. t. } \frac{4\pi d^2 n_i \cos\theta_i}{\lambda^2} \geq h(c_i, \alpha, \Gamma_i, L_i, p_t), \forall i, \tag{9}$$

$$\frac{\lambda}{d n_i \cos\theta_i \delta_{ij}} + \frac{\lambda}{d n_j \cos\theta_j \delta_{ij}} \leq 1, \forall i, j, i \neq j, \tag{10}$$

$$\max_{i,j}\left\{\frac{(n_i+1)\cos\theta_i}{n_j \cos\theta_j}\right\} \leq \xi, \forall i, j, i \neq j, \tag{11}$$

$$n_i \geq l, \forall i, \tag{12}$$

$$\sum_i n_i \leq N - K, \tag{13}$$

$$n_i > 0, \forall i. \tag{14}$$

Introducing the auxiliary variable x, the above optimization problem is equivalent to

$$\min_{n_i, x} \prod_i \left\{\frac{d\pi n_i \cos\theta_i}{2.78\lambda}\frac{\pi}{180}\right\} \tag{15}$$

$$\text{s. t. } (9), (10), (12), (13), (14), \tag{16}$$

$$\frac{x}{\xi} \leq 1, \tag{17}$$

$$\frac{(n_i+1)\cos\theta_i}{x n_j \cos\theta_j} \leq 1, \forall i, j, i \neq j. \tag{18}$$

This optimization problem is a geometric optimization problem [20], and can be efficiently solved. We denote its solution as $\hat{n}_i$.

B. Rounding Method

Since the solution $\hat{n}_i$ cannot satisfy the integer constraint (7), we take the following strategy to round it to the integer.

$$\tilde{n}_i = \text{floor}(\hat{n}_i) + 1, \tag{19}$$

where $\text{floor}(y) = \max_{\tilde{y} \in \mathbb{N}}(\tilde{y} \leq y)$.

C. Feasibility Analysis

The feasibility of our beam management scheme is as follows.

Theorem 1: Using the rounding method in (19), the solution $\tilde{n}_i$ can satisfy constraints (2)-(7).

Proof: See Appendix A. ■

D. Network Control Strategy

Based on the above, we summarize our beam management scheme in mmWave network as in Alg. 1.

Algorithm 1: Beam Management Scheme

Output: Beam Management Decision $\tilde{n}_i$

1. Obtain the steering direction θ_i and the beamforming gain requirement $h(c_i, \alpha, \Gamma_i, L_i, p_t)$ in the network association process;
2. Solve the optimization problem (15)-(18) to derive $\hat{n}_i$;
3. Derive the beam configuration decision $\tilde{n}_i$ using the rounding method in (19);
4. Configure the beam using element number $\tilde{n}_i$ and steer the beam towards direction of θ_i for serving user i;

IV. Simulation Results and Performance Evaluation

In this section, we evaluate our proposed beam management scheme through simulation. We used CVX, a package for specifying and solving convex programs [21], to solve our relaxed beam management problem (15)-(18). We compare our algorithm with the optimal solution, which is derived by the exhaustive search method.

A. Simulation Setup

We evaluate our proposed algorithm in an mmWave network, in which an ULA is deployed over AP with the sum element number $N = 256$. We set the antenna element spacing $d = \frac{\lambda}{2}$. We further suppose that 6 users are randomly located around the AP, and each user demands certain amount beamforming gain from 9dB to 15dB, to compensate for the severe signal attenuation. The steering direction is chosen from $(-\frac{\pi}{2}, \frac{\pi}{2})$. Moreover, we set the element number $n_i \geq 10$ so as to guarantee that the SLL is greater than 12dB. We also let $\xi = 4$ to ensure that the main-lode gain difference is smaller than 6dB.

B. Performance Analysis

We first compare the performance of our proposed beam management algorithm with the exhaustive search method. We run our simulation in 7 different configurations, where the steering direction and the beamforming gain are randomly set. We can see from Fig. 5 that the performance of our proposed method is approaching the exhaustive search method. The maximum performance gap is 1.2%, but our algorithm reduces the computational complexity significantly.

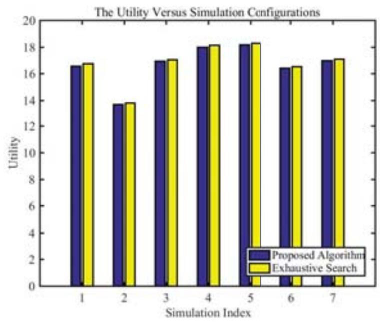

Fig. 5. Performance Gap Under Different Simulation Configurations

TABLE I
First Null Beamwidth (deg) for Each User

User	1	2	3	4	5	6
Proposed	7.84	29.10	13.48	12.06	13.09	7.78
Exhaustive	9.81	31.75	12.73	14.11	14.73	5.91

Moreover, we further look into the first null beamwidth in the fifth simulation. The minimum angle difference is 7.89deg between user 1 and user 6. We can see from the Tab. I that the angle difference is $\frac{7.84+7.78}{2} = 7.81$deg and $\frac{9.81+5.91}{2} = 7.86$deg for our proposed algorithm and the exhaustive search method, which are both smaller than the minimum user angle difference 7.89deg, thus satisfying the main-lobe interference constraint.

V. Conclusions

In this paper, we study the beam management problem of the ULA based mmWave network in a systematic perspective regarding the user mobility, link outage, and interference control. We optimize the beam coverage to better support the user mobility. To compensate for the severe signal attenuation in mmWave band, we configure the beam to reduce the link outage probability. Further, we control the interference from both the main-lobe and the side-lobe. We formulate this beam management problem as a non-linear integer optimization problem. We deliberately design its solution, which has a low computational complexity. We verify our proposed algorithm through simulation, and simulation results demonstrate that our beam management scheme can efficiently serve multiple mobile users in mmWave network.

Appendix A
Proof of Theorem 1

We analyze the feasibility of our rounding strategy in this appendix. From the rounding scheme of (19), we can derive that $\tilde{n}_i \geq \hat{n}_i$. Since $\hat{n}_i$ satisfies constraint (9), (10), and (12), it can easily derive that $\hat{n}_i$ satisfies (2), (3), and (5).

Further, since $\hat{n}_i$ satisfies constraint (11), we can derive that

$$\hat{n}_i \cos\theta_i - \hat{n}_j \xi \cos\theta_j \leq -\cos\theta_i, \forall i, j. \tag{20}$$

For the integer solution $\tilde{n}_i$, we can find a constant $0 < \rho_i \leq 1$ such that $\tilde{n}_i = \hat{n}_i + \rho_i$. Then, for $\forall i, j$, we can derive

$$\begin{aligned} & \tilde{n}_i \cos\theta_i - \tilde{n}_j \xi \cos\theta_j \\ =& (\hat{n}_i + \rho_i)\cos\theta_i - (\hat{n}_j + \rho_j)\xi \cos\theta_j \\ =& \hat{n}_i \cos\theta_i - \hat{n}_j \xi \cos\theta_j + \rho_i \cos\theta_i - \rho_j \xi \cos\theta_j \\ \leq& (\rho_i - 1)\cos\theta_i - \rho_j \xi \cos\theta_j \\ \leq& -\rho_j \xi \cos\theta_j < 0. \end{aligned} \tag{21}$$

Therefore, $\forall i, j$, we have $\tilde{n}_i \cos\theta_i < \tilde{n}_j \xi \cos\theta_j$. Then we have $\max_{i,j}\{\frac{\tilde{n}_i \cos\theta_i}{\tilde{n}_j \cos\theta_j}\} \leq \xi, \forall i, j$, which means constraint (4) is satisfied.

Further, from (13), we have $\sum_i \hat{n}_i \leq N - K$. Hence, we can derive

$$\sum_i \tilde{n}_i = \sum_i (\hat{n}_i + \rho_i) = \sum_i \hat{n}_i + \sum_i \rho_i \leq N - K + K = N.$$

Thus, $\tilde{n}_i$ can satisfy (6).

Therefore, the beam management $\tilde{n}_i$ can satisfy constraints (2)-(7).

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