Transportation Research Part B 123 (2019) 64–87

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

# A multi-stage stochastic programming model for relief distribution considering the state of road network

Shaolong Hu[a,b], Chuanfeng Han[a], Zhijie Sasha Dong[b], Lingpeng Meng[c,*]

[a] School of Economics and Management, Tongji University, 1239 Siping Road, Shanghai 200092, China
[b] Texas State University, 601 University Drive, San Marcos, TX 78666, United States
[c] China Institute of FTZ Supply Chain, Shanghai Maritime University, 1550 Haigang Avenue, Shanghai 201306, China

ARTICLE INFO

*Article history:*
Received 4 June 2018
Revised 13 February 2019
Accepted 19 March 2019

*Keywords:*
Emergency logistics
Transportation
Uncertain and dynamic road capacity
Multi-stage stochastic programming
Progressive hedging algorithm

ABSTRACT

As an important aspect in disaster operations management, relief distribution has been challenged by lots of factors, such as unpredictable occurrence time, intensity and location of secondary disasters (e.g. aftershocks and landslides, which usually occur after an earthquake), and availability of vehicles. A multi-stage stochastic programming model is developed for disaster relief distribution with consideration of multiple types of vehicles, fluctuation of rental, and the state of road network. The state of road network is characterized using uncertain and dynamic road capacity. The scenario tree is employed to represent the uncertain and dynamic road capacity, and demonstrate the decision process of relief distribution. A progressive hedging algorithm (PHA) is proposed for solving the proposed model in large-scale size. Based on a real-world case of Yaan earthquake in China, numerical experiments are presented to study the applicability of the proposed model and demonstrate the effectiveness of the proposed PHA. Useful managerial insights are provided by conducting numerical analysis.



## 1. Introduction

Large amounts of relief supplies, such as water, tents, medical supplies, and fuel, are required in the aftermath of a major disaster. Satisfying victims' needs is crucial to the success of disaster relief operations, as a lack of relief supplies may cause suffering and life loss for victims (Ahmadi et al., 2015). Relief supplies are usually insufficient due to damages to local inventory and markets. Therefore, procuring relief supplies from distant locations, and transporting them to the disaster-affected areas within a given time frame is of great importance. The aim of this paper is to propose an efficient distribution plan to provide prompt responses to victims' demand for relief supplies.

The relief distribution plan is a major component in disaster operations management (Balcik et al., 2010). Decisions include the design of the emergency supply network (Sheu and Pan, 2014; Meng et al., 2017), locating distribution centers (Zokaee et al., 2016), vehicle routing (Rennemo et al., 2014) and last mile distribution (Lu et al., 2016; Zhou et al., 2017). These work usually assume that no limitation exists in the availability of vehicles or the state of road network. However, the distribution of relief supplies is highly dependent on the capacity of available vehicle fleets. Moreover, relief distribution decisions may not work due to a collapsed network with totally or partially blocked roads. It is useless to store sufficient

* Corresponding author.
*E-mail address:* lmeng@tongji.edu.cn (L. Meng).

https://doi.org/10.1016/j.trb.2019.03.014
0191-2615/

relief supplies if they cannot be effectively delivered to the affected location. Thus, it is essential to integrate fleet decisions and the state of road network into relief distribution.

There are two challenges in relief distribution of current common disaster practice. (1) The number of vehicles and costs for using them vary. We use "vehicles" in this paper to represent all land and air vehicles, such as airplanes, trains, trucks, and helicopters. Relief agencies usually do not own or operate vehicle fleets, and typically rent vehicles and drivers for delivering relief supplies (Balcik et al., 2010). Vehicle are usually insufficient when there is a sudden increase in the need for delivering supplies in post-disaster situations, which will increase the rental cost of vehicles (Balcik et al., 2010). (2) The state of road network also vary. The occurrence time, intensity and location of secondary disasters are unpredictable, which may cause variation of road capacity within the affected region (the detailed explanations are shown in the next paragraph). It may make the use of vehicles challenging. Motivated by the significance of developing relief distribution plans, we study a relief distribution problem with consideration of multiple types of vehicles, fluctuation of rental, and the state of road network to quickly respond to natural disasters.

When a major earthquake occurs, several segments of roads are damaged in a local network and would decrease road capacity. For instance, assuming a road has a capacity of passing 100 vehicles per hour. Depending on the degree of damage to the road, the capacity could reduce to 50, 30 or 10 per hour. Obviously, the corresponding capacity equals to zero only if a road is disrupted. For quicker casualty transfer and relief supply delivery, road recovery should be performed as soon as possible to improve the road capacity. However, secondary disasters might continue to damage recovered or recovering segments of the road network. Secondary disasters, such as aftershocks, mountain collapse, landslides, debris flows, and flooding caused by dam breaching, usually follow major earthquakes (Zhang et al., 2012), especially in mountainous areas. 64 and 104 major aftershocks, ranging in magnitude from 4.0 to 6.1, were recorded within 72 h of the Wenchuan earthquake (China Earthquake Administration, 2008). The road capacity might become uncertain and dynamic because the intensity and location of secondary disaster is unpredictable and secondary disaster will be continually occur. The uncertainty and dynamic of road network capacity increase the complexity of relief distribution. In this study, a multi-stage stochastic programming approach is employed to formulate the problem. More specific explanations about uncertain and dynamic road network capacity and decisions of multi-stage relief distribution are depicted in Section 3.1.

Relief supplies are transported to the disaster region and stored in selected temporary warehouses. Then, the "last mile distribution" has to be carried out according to the differential demands of disaster-affected locations. Motivated by the relief practice, we consider relief distribution as a transshipment problem. The transshipment problem is formulated as a multi-stage stochastic programming model with consideration of multiple types of vehicles, fluctuation of vehicle rental cost, and the state of road network. A scenario-based approach is applied to represent the uncertain and dynamic road capacity, wherein a scenario tree is employed to demonstrate the decision process of multi-stage relief distribution. As the multi-stage stochastic programming models are difficult to be solved and time-consuming even for small-sized instances, we propose a progressive hedging algorithm for solving the proposed model. Based on real-world case data, numerical experiments are performed to study the applicability of our model and explore its managerial implications for disaster relief distribution. An appropriate distribution plan can be especially beneficial in terms of saving costs, utilizing road network, and satisfying victims' needs.

This paper is organized as follows: Section 2 presents related studies in multiple types of vehicles, the state of road network and the progressive hedging algorithm. Section 3 develops a multi-stage stochastic programming model. Section 4 describes the implementation of the progressive hedging algorithm. Section 5 describes a case study, while in addition, findings are observed from the numerical results, and managerial implications are summarized. Section 6 provides conclusions and summary.

## 2. Literature review

(1) Multiple types of vehicles

Vehicles are used to transport personnel, aid items, and materials. Without vehicles, there is no aid (Wassenhove and Martinez, 2012). To make relief supplies distribution decisions more practical, vehicle capacity constraints (Mete and Zabinsky, 2010; Sawik, 2014) and vehicle routing problem (Dong and Turnquist, 2015; Ahmadi et al., 2015; Dong, 2015) are commonly considered in the literature. These work focused on truck schedules to deliver supplies, and have ignored the issue of road availability in the relief distribution. For instance, trucks cannot be used if road links have been disrupted. Ferrer et al. (2018) addressed a vehicle schedule problem in last mile distribution. Although three types of trucks with different capacity and speed are considered, in essence, they considered one type of vehicle.

Considering multiple ways of accessing affected locations is beneficial to increase the efficiency of relief distribution. For instance, helicopters are usually used for delivering relief supplies and injured people if areas are not accessible due to disruption of links (Barbarosoğlu et al., 2002; Wassenhove, 2006). Liu et al. (2018) studied a multi-commodity, multi-period distribution problem that considers helicopters for quick delivering relief commodities and injured people. To the best of our knowledge, there are few works that considered the use of multiple types of vehicles. Alem et al. (2016) developed a two-stage stochastic network flow model for decisions on the fleet size of multiple types of vehicles over a dynamic multiperiod horizon. Moreno et al. (2016) developed a two stochastic mixed-integer programming model to integrate and coordinate facility location, transportation, and fleet sizing decisions. These two work all considered trucks, boats, and helicopters to deliver relief supplies. However, they ignored fluctuation of rental for vehicles. The sudden surge of demand may inflate the rental costs of available vehicles (Balcik et al., 2010), which might affect on the related costs and transportation decisions.

(2) The state of road network

The state of road network is usually considered as a stochastic element in the relief distribution models. Most of the research associated the cost, distance or time of transportation to indirectly reflect the state of road network. Balcik and Beamon (2008) associated the travel costs on arcs with different scenarios. According to their approach, the unit transportation cost on the arcs can be assigned a large number when they are damaged or cannot be used by a specific vehicle type. Baskaya et al. (2017) used different route distance between relief centers and affected locations to reveal the disruption levels of the network. Tofighi et al. (2016), and Elçi and Noyan (2018) presented the state of the road network through different transportation time in different disaster scenarios. The disruption levels are reflected by transportation time. Their work reveals that depending on the disruption levels, longer transportation time is needed than usual due to the decrease of arc capacity. In other words, smaller number of vehicles can be traversed on an arc than usual in the same time period when arc capacity decreases.

Alem et al. (2016), Moreno et al. (2016), and Ferrer et al. (2018) used binary variables to describe the state of arcs in the relief distribution problem. In their approach, if an arc cannot be used by a specific vehicle type, the parameter of the state of arcs equals zero, otherwise, it equals one. The vehicle-road compatibility (i.e., binary variables) is reflected in a set of scenarios. It is unpractical for just consider states of arcs as non-disruption and disruption. To the best of our knowledge, there is one work which used integer variables to describe the state of arcs (Rennemo et al., 2014). According to their approach, arc capacity denoted the maximum number of vehicles that can traverse on an arc. The capacity equals to zero only if an arc is disrupted. It is apparent that integer variables can better represent the various state of arcs than binary variables. In sum, the existing work that employ integer variables to describe the state of arcs appear to be scarce. Moreover, the existing work ignored the variation of road capacity over time. As discussed in Introduction, the road capacity might become dynamic because of secondary disaster will be continually occur. Table 1 shows the summary of most relevant work on multiple types of vehicles and the state of road network.

(3) Progressive hedging algorithm

For solving large-scale problems efficiently, the progressive hedging algorithm (PHA) is adopted. Studies for progressive hedging algorithm can be divided into three categories. The first category is the application of PHA in various fields such as supply chain management (Kim et al., 2015; Dong et al., 2018), health care (Gul et al., 2015) and harvest planning (Veliz et al., 2015). Another category is PHA enhancements through methods of adjusting penalty parameters. Listes and Dekker (2005) discussed the sensitivity of the performances of PHA to the changes of the penalty parameter. Hvattum and Lokketangen (2009) proposed a method for comparing the convergence rate at iterations $k$ and $k$-1, increasing the penalty parameter if the convergence rate is decreasing. The penalty parameter is decreased if the current status is closer to consensus variables at iteration $k$-1 than at iteration $k$. Watson and Woodruff (2011) developed a method to adjust penalty parameters to enhance PHA for scenario-based resource allocation problems. The third category is to integrate other algorithms (e.g., grouping method, dual decomposition and sample average approximation) into PHA to efficiently solve scenario specific sub-problems. Crainic et al. (2014) proposed a $k$-means based approach for grouping scenarios. They evaluated these strategies in the context of stochastic network design by analyzing the performance of a new progressive hedging-based meta-heuristic. Guo et al. (2015) presented a method for integrating the PHA and dual decomposition algorithm for stochastic mixed-integer programs. Dong et al. (2015) integrated a sample average approximation method and PHA for joint decision-making of shipping service capacity planning and dynamic container routing. To the best of our knowledge, the existing studies all considered binary variables so that quadratic penalty components can be easily linearized. Moreover, PHA for multi-stage stochastic programming models seems to be lacking.

Here, our study aims to efficiently integrate two strategies into PHA for solving the proposed multi-stage stochastic programming model. One strategy is to linearize the quadratic penalty components which contain the integer decision variables. Another is the variables fixing strategy, which force binary variables equal to 1 if the value of corresponding consensus variables is larger than a threshold. The details are addressed in Section 4.

In summary, our work features the following contributions to the disaster relief operations literature.

(1) A large amount of vehicles are required for delivering relief supplies due to the sudden surge of demand. The vehicles may thus be scarce. The existing work that consider the use of multiple types of vehicles and fluctuation of rental cost appear to be lack. Therefore, a quantity- and time interval-based rental for hiring four types of vehicles (i.e., airplane, train, truck and helicopter) is constructed to study its impact on relief distribution decisions. This study can help relief agencies in making contractual agreements with carriers in the disaster preparedness stage and improve the accuracy in determining an annual emergency budget.
(2) In many real-world cases, the road capacity is usually uncertain and dynamic because the intensity and location of secondary disaster are unpredictable and secondary disaster will continually occur. The existing work ignored the variation of road capacity over time. Therefore, we analyze the impact of secondary disasters on road network capacity. The state of road network (i.e., uncertain and dynamic road capacity) is modeled through the scenario tree.

**Table 1**
Summary of most relevant work on multiple types of vehicles and the state of road network.

| | Vehicle features | | | | | | Uncertain road network treatment | | | Modeling | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Types | | Available quantity | | Rental cost | | Stoch. road capacity | | Stoch. cost/ time/distance | Two-stage | Multi-stage |
| | Single | Multiple | Fixed | Variable | Fixed | Variable | Binary | Integer | | | |
| Balcik and Beamon (2008) | | | | | | | | | √ | √ | |
| Rennemo et al. (2014) | √ | | √ | | | | | √ | | √ | |
| Alem et al. (2016) | | √ | √ | | √ | | √ | | | √ | |
| Moreno et al. (2016) | | √ | √ | | √ | | √ | | | √ | |
| Tofighi et al. (2016) | | | | | | | | | √ | √ | |
| Baskaya et al. (2017) | | | | | | | | | √ | | |
| Ferrer et al. (2018) | √ | | √ | | √ | | √ | | | | |
| Elçi and Noyan (2018) | | | | | | | | | √ | √ | |
| Liu et al. (2018) | √ | | | √ | | | | | | | |
| This paper | | √ | | √ | | √ | | √ | | | √ |

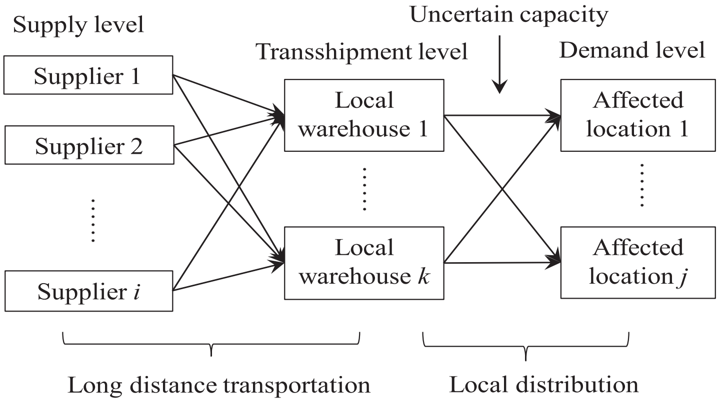


**Fig. 1.** A transshipment network in relief distribution.

Incorporating the state of road network is beneficial for improving the accuracy of decision making in relief distribution.

(3) The existing work formulated relief distribution problems as two-stage stochastic programming models. These models are static in nature because of the decisions are made at one point in time. Therefore, the relief distribution problem with consideration of uncertain and dynamic road capacity is formulated as a multi-stage stochastic programming model. The proposed model can help relief agencies to produce plans for dynamic vehicle allocation and relief supplies distribution.

## 3. Modeling

### 3.1. Problem description

Based on disaster relief practices, a transshipment network in relief distribution can be depicted as Fig. 1. This transshipment network consists of the supply level, transshipment level, and demand level. The supply level comprises several suppliers serving as replenishment resources. The transshipment level comprises warehouses or distribution centers. Relief supplies may experience multiple transshipment actions before arriving at affected locations. We consider one transshipment level located in the disaster-affected region. The demand level consists of affected locations with diverse needs. Relief distribution can thus be roughly divided into two phases. The first phase is called long distance transportation. In this phase, relief supplies are primarily transported via air and rail transportation. The next phase is local distribution, and relief supplies are delivered to affected locations mainly via trucks and helicopters. As the impact of secondary disasters on road recovery, we consider the road network capacity for delivering relief supplies to be uncertain and dynamic in the local distribution phase.

Considering the time required for long distance transportation and road recovery, the proposed model focuses on daily decisions for the dynamic allocation of vehicles and distribution of relief supplies over a finite planning horizon. We consider airplanes and trains for long distance transportation, and trucks and helicopters for local distribution. Inspired by relief practices, we consider the following specific situation. For long distance transportation, large quantities of supplies are transported to safe airports and train stations, and then, trucks are used to transport relief supplies to local warehouses. Relief supplies are usually collected from other provinces (that are hundreds of kilometers away) and delivered with long distance transportation. Therefore, the difference in transport speed between airplanes and trains is considered. We assume that an additional day is required if trains are used for transporting relief supplies. For local distribution, we consider that supplies are delivered from local warehouses to several fixed points, which are easily accessible by trucks and helicopters.

A variety of resources is required in the affected regions. There are not only distribution of relief supplies, but also vehicle flows for evacuation and rescue equipment and other operations in road network. Moreover, since road conditions are uncertain (e.g. partially or fully damaged), the road capacity for relief distribution is usually limited. Therefore, the number of traversable vehicles (i.e. the quantity of relief items) per time interval during post-disaster is smaller than normal situation. We introduce a scenario tree to depict the uncertain and dynamic road capacity. A sample of the scenario tree with three stages is identified in Fig. 2. Each stage denotes a time interval. Depending on the specific problem, a time interval could be hours, days, or even months. As we consider daily decisions, each stage denotes one day. The scenario tree comprises a number of nodes and arcs. Each node denotes a possible realization (named scenario, indexed by $s \in S$) of the road capacity with a specific probability. The root scenario of the tree represents the current road capacity. The arcs denote the direction of scenarios for the next stage. A probability is associated with each arc denoting the probability of the corresponding scenario. Note that the probability of scenario $s$ equals to the product of the probability of

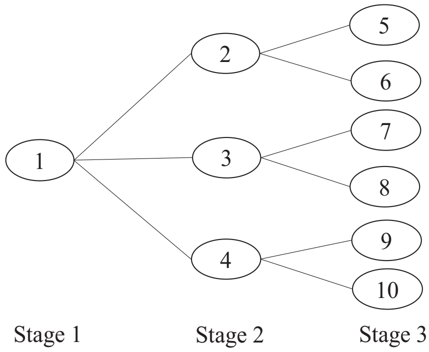


**Fig. 2.** A sample of the scenario tree.

the arcs from the root scenario to scenario $s$, and the sum of probabilities of scenarios at each stage is 1 (Zanjani and Nourelfath, 2014). The specific steps for estimating the road capacity of scenarios and its probability are shown in Section 5.1.

In the sample scenario tree, scenario 1 is the root scenario, scenarios 2–4 belong to stage 2 and scenarios 5–10 belong to stage 3. Fig. 2 illustrates the road capacity under scenario 5 and 6 varies based on the state of scenario 2 and the impact of secondary disasters on the roads under scenarios 5 and 6. The same relationship exists for scenarios (1, 2, 3 and 4), (3, 7 and 8) and (4, 9 and 10). Therefore, we introduce a set of parent scenarios (indexed by $\xi(s) \in \Omega$) to represent this relationship. For instance, scenario 2 is the parent scenario of scenarios 5 and 6 where $\xi(5)= 2$ and$\xi(6)= 2$. As shown in Fig. 2, the set of parent scenarios $\Omega$ comprises 4 elements.

We formulate the above-mentioned transshipment problem as a multi-stage stochastic programming model. The dynamic vehicle allocation and relief supply distribution are explained as follows. In the first stage (i.e. the first day), there is usually one scenario (e.g., scenario 1 in Fig. 2), and we assume that demand is 0. According to the predicted probability and the state of road network of scenarios for subsequent stages (e.g., scenarios 2–10), locations and inventory of warehouses and the maximal quantity of vehicles for the next stage should be determined. Then, at the beginning of next stage, depending on the actual road capacity of scenarios (e.g., scenarios 2–4) and maximal quantity of vehicles (which is determined in the previous stage), vehicles are allocated to deliver supplies from suppliers to warehouses, then to affected areas. Furthermore, at the end of each stage (e.g., scenarios 2–4), the maximum quantity of vehicles for the next stage should be updated. Note that the locations of warehouses are fixed after the first stage.

### *3.2. Model formulation*

Assumptions, definitions of sets, parameters, and decision variables are given below.

**Assumptions**

(1) The long distance transportation network is unaffected by the disaster because 1) the network is relatively far away from the disaster-affected region, and 2) warehouses are usually located at strategic locations which are accessible via multiple routes.
(2) Detour is not considered because multiple routes between two locations may not always exist (e.g., mountainous areas). Helicopters are used for local distribution, or ground transportation can be carried out after damaged segments of roads are repaired.
(3) We assume that the demand for supplies is steady. This is because aftershocks and landslides usually damage recovered or recovering segments of the roads.

**Sets and indices**

| | |
|---|---|
| $I$ | Set of suppliers, indexed by $i \in I$. |
| $K$ | Set of warehouses, indexed by $k \in K$. |
| $J$ | Set of affected locations, indexed by $j \in J$. |
| $A$ | Set of items types, indexed by $a \in A$. |
| $O$ | Set of vehicle types (i.e. airplane and train) used for long distance transportation, indexed by $o \in O$. |
| $P$ | Set of vehicle types (i.e. helicopter and truck) used for local distribution, indexed by $p \in P$. |
| $L$ | Set of quantity intervals, indexed by $l \in L$. |
| $S$ | Set of scenarios, indexed by $s \in S$. |
| $\Omega$ | Set of parent scenarios, indexed by $\xi(s) \in \Omega$. |

**Parameters**

$\varphi_{k,j,s}$ Capability of the road from warehouse $k$ to affected location $j$ under scenario $s$.
$\omega_s$ Probability of scenario $s$.
$D_{a,j}$ Demand of disaster-affected location $j$ for type $a$ items.
$HS_{i,k}$ Distance from supplier $i$ to warehouse $k$.
$HW_{k,j}$ Distance from warehouse $k$ to disaster-affected location $j$.
$Q_{k,a}$ Handling capacity of warehouse $k$ for type $a$ items.
$R_{k,a}$ Inventory of type $a$ items at warehouse $k$.
$U_{i,a}$ Inventory of type $a$ items at supplier $i$.
$\mu_a$ Volume of type $a$ items.
$CH_a$ Unit handling cost for type $a$ items.
$G_a$ Unit penalty cost for the shortage of type $a$ items.
$VS_{o,l}$ Maximal number of type $o$ vehicles in quantity interval $l$.
$VW_{p,l}$ Maximal number of type $p$ vehicles in quantity interval $l$.
$ES_o$ Transport capacity of type $o$ vehicles.
$EW_p$ Transport capacity of type $p$ vehicles.
$CRS_{o,l,s}$ Unit cost for renting type $o$ vehicles in quantity interval $l$ under scenario $s$.
$CRW_{p,l,s}$ Unit cost for renting type $p$ vehicles in quantity interval $l$ under scenario $s$.
$CTS_{o,a}$ Unit transport cost for items of type $a$ items via type $o$ vehicles.
$CTW_{p,a}$ Unit transport cost for items of type $a$ items via type $p$ vehicles.
$\gamma$ Number of selected warehouses.
$\eta$ The number of the first scenario at the third stage (e.g. $\eta$=5, see Fig. 2).
$M$ Large positive number.

**Decision variables**

$w_k$ If warehouse $k$ is selected for temporarily storing supplies, $w_k = 1$; otherwise, $w_k = 0$.
$xs_{o,l,s}$ If type $o$ vehicles is set to the quantity interval $l$ under scenario $s$, $xs_{o,l,s} = 1$; otherwise, $xs_{o,l,s} = 0$.
$xw_{p,l,s}$ If type $p$ vehicles is set to the quantity interval $l$ under scenario $s$, $xw_{p,l,s} = 1$; otherwise, $xw_{p,l,s} = 0$.
$ys_{i,o,l,s}$ Number of type $o$ vehicles used by supplier $i$ in quantity interval $l$ under scenario $s$.
$yw_{k,p,l,s}$ Number of type $p$ vehicles used by warehouse $k$ in quantity interval $l$ under scenario $s$.
$y_{k,j,p,s}$ Number of type $p$ vehicles used for delivering supplies from warehouse $k$ to affected location $j$ under scenario $s$.
$qs_{i,k,o,a,s}$ Quantity of type $a$ items delivered from supplier $i$ to warehouse $k$ via type $o$ vehicles under scenario $s$.
$qw_{k,j,p,a,s}$ Quantity of type $a$ items delivered from warehouse $k$ to affected location $j$ via type$p$ vehicles under scenario $s$.
$v_{k,a,s}$ Quantity of type $a$ items at warehouse $k$ under scenario $s$.
$h_{k,a,s}$ Additional quantity of type $a$ items which need be handled at warehouse $k$ under scenario $s$.
$z_{j,a,s}$ Shortage of type $a$ items at affected location $j$ under scenario $s$.

Based on the above-mentioned descriptions and definitions, a multi-stage stochastic programming model is formulated with the objectives of minimizing rental cost, transportation cost, handling cost, and penalty cost. The total expected costs are computed with Eq. (1). Rental cost ($rc$) is the costs of renting vehicles, as computed with Eq. (2). Transportation cost ($tc$) is a function of unit transportation cost and delivered quantity, as computed with Eq. (3). The temporary warehouses exist for at most 10–30 days after disasters and are usually located at cooperating suppliers. The costs for locating temporary warehouses are insignificant. However, in practice, with the exception of a scheduled distribution plan, the in-kind donations are delivered to disaster-affected provinces via highway transportation. There may be insufficient space and workforce capable of storing and handling all the supplies. This imposes great challenges on local warehouses. Therefore, the model includes handling cost ($hc$), which is a function of unit handling cost and the quantity of items that need to be handled by an additional workforce, as computed with Eq. (4). Penalty cost ($pc$) is a function of unit penalty cost and unsatisfied demand, as computed with Eq. (5).

$$f = \min \sum_s \omega_s \cdot (rc_s + tc_s + hc_s + pc_s) \tag{1}$$

$$rc_s = \sum_{i,o,l} \left( ys_{i,o,l,s} \cdot CRS_{o,l,s} \right) + \sum_{k,p,l} \left( yw_{k,p,l,s} \cdot CRW_{p,l,s} \right), \quad \forall s \tag{2}$$

$$tc_s = \sum_{i,k,o,a} \left( CTS_{o,a} \cdot HS_{i,k} \cdot qs_{i,k,o,a,s} \right) + \sum_{k,j,p,a} \left( CTW_{p,a} \cdot HW_{k,j} \cdot qw_{k,j,p,a,s} \right), \quad \forall s \tag{3}$$

$$hc_s = \sum_{k,a} CH_a \cdot h_{k,a,s}, \quad \forall s \tag{4}$$

$$pc_s = \sum_{j,a} G_a \cdot z_{j,a,s}, \quad \forall s \tag{5}$$

The first group of constraints is to locate the warehouses. Eq. (6) restricts the number of selected warehouses. Eq. (7) restricts the use of vehicles $p$ to the selected warehouses.

$$\sum_{k} w_k \leq \gamma \tag{6}$$

$$w_k \leq \sum_{p,l} yw_{k,p,l,s} \leq M \cdot w_k, \quad \forall k, s \tag{7}$$

The second group of constraints is to allocate vehicles. Eqs. (8) and (9) restrict that at most one quantity interval of vehicle types that can be selected under each scenario. We discretize the continuous number of vehicles as the quantity interval. The higher the quantity interval hired, the more specific the number of vehicles that can be used. For example, 20 vehicles are available if quantity interval 1 is hired, and 40 vehicles are available if quantity interval 2 is hired. Eqs. (10)–(12) restrict the number of vehicles to suppliers, warehouses, and the affected locations, respectively. We add the restriction to the number of vehicles used under current scenarios to be less than the quantity of available vehicles under its corresponding parent scenario.

$$\sum_{l} xs_{o,l,s} \leq 1, \quad \forall o, s \tag{8}$$

$$\sum_{l} xw_{p,l,s} \leq 1, \quad \forall p, s \tag{9}$$

$$xs_{o,l,\xi(s)} \leq \sum_{i} ys_{i,o,l,s} \leq xs_{o,l,\xi(s)} \cdot VS_{o,l}, \quad \forall o, l; s \geq 2 \tag{10}$$

$$xw_{p,l,\xi(s)} \leq \sum_{k} yw_{k,p,l,s} \leq xw_{p,l,\xi(s)} \cdot VW_{p,l}, \quad \forall p, l; s \geq 2 \tag{11}$$

$$\sum_{j} y_{k,j,p,s} = \sum_{l} yw_{k,p,l,s}, \quad \forall k, p; s \geq 2 \tag{12}$$

The third group of constraints is flow constraints. Eqs. (13)–(18) restrict the quantity delivered via vehicles to be less than its corresponding transport capacity. Note that there are 2 elements in sets of $O$. We use 1 and 2 to represent airplanes and trains, respectively. Considering the transport speed of trains, we thus add the restriction that the quantity transported under current scenarios to be less than the total transport capacity of trains employed under the corresponding parent scenarios. For instance, a quantity interval of trains is selected under the first stage of the scenario. The number of trains used is then determined and the relief supplies loaded accordingly, under the second stage of the scenario. The relief supplies then arrive at the warehouses under the third stage of the scenario. There are no such restrictions for airplanes, trucks, or helicopters. Eq. (19) adds the restriction so that transported quantity from a supplier to all warehouses should not exceed its available quantity. Eq. (20) restricts the delivered quantity via truck to be less than the network capacity. Note that there are 2 elements in the set of $P$, where 1 and 2 represent trucks and helicopters, respectively. Although network capacity is related to the quantity of items by Eq. (20), the quantity of items is also restricted by the number of vehicles, as shown by Eq. (18), which means vehicles are related to the capacity, i.e., according to the network capacity, the model will assign an optimal number of vehicles for delivering relief supplies.

$$qs_{i,k,1,a,1} = 0, \quad \forall i, k, a \tag{13}$$

$$qs_{i,k,2,a,s} = 0, \quad \forall i, k, a; s < \eta \tag{14}$$

$$\sum_{k,a} qs_{i,k,1,a,s} \cdot \mu_a \leq \sum_{l} ys_{i,1,l,s} \cdot ES_1, \quad \forall i; s \geq 2 \tag{15}$$

$$\sum_{k,a} qs_{i,k,2,a,s} \cdot \mu_a \leq \sum_{l} ys_{i,2,l,\xi(s)} \cdot ES_2, \quad \forall i; s \geq \eta \tag{16}$$

$$qw_{k,j,p,a,1} = 0, \quad \forall k, j, p, a \tag{17}$$

$$\sum_{a} qw_{k,j,p,a,s} \cdot \mu_a \leq y_{k,j,p,s} \cdot EW_p, \quad \forall k, j, p; s \geq 2 \tag{18}$$

$$\sum_{k,o} qs_{i,k,o,a,s} \leq U_{i,a}, \quad \forall i, a, s \tag{19}$$

$$\sum_{a} qw_{k,j,1,a,s} \cdot \mu_a \leq \varphi_{k,j,s}, \quad \forall k, j, s \tag{20}$$

The fourth group of constraints is to describe the inventory of warehouses. Considering that a certain quantity of supplies is usually stored in each province for emergency events, the quantity of supplies at warehouses are calculated with Eqs. (21) and (22) under each scenario. Because the handling capacity and space of warehouses is limited, costs are generated by handling the additional quantity of supplies, calculated with Eq. (23).

$$v_{k,a,1} = R_{k,a} \cdot w_k, \quad \forall k, a \tag{21}$$

$$v_{k,a,s} = v_{k,a,\xi(s)} + \sum_{i,o} qs_{i,k,o,a,s} - \sum_{j,p} qw_{k,j,p,a,s}, \quad \forall k, a; s \geq 2 \tag{22}$$

$$h_{k,a,s} \geq \sum_{i,o} qs_{i,k,o,a,s} + \sum_{j,p} qw_{k,j,p,a,s} - Q_{k,a}, \quad \forall k, a, s \tag{23}$$

The last group of constraints calculates unsatisfied demand and handles the boundaries and integrities of decision variables. Set $A$ consists of 3 elements. Type 1 items are those that are consumed regularly, such as food, water, and hygiene kits. Supplies like tents, blankets, coat, lighting equipment, and mosquito nets are examples of Type 2 items. Type 3 items are medical supplies. For those supplies which are consumed regularly (i.e. type 1 items), the unsatisfied demand is calculated with Eq. (25). For type 2 and 3 items, unsatisfied demand is calculated with Eqs. (26) and (27). Eqs. (28)–(31) defines the integrity of the variables.

$$z_{j,a,1} = 0, \quad \forall j, a \tag{24}$$

$$z_{j,1,s} = D_{j,1} - \sum_{k,p} qw_{k,j,p,1,s}, \quad \forall j; s \geq 2 \tag{25}$$

$$z_{j,a,s} = D_{j,a} - \sum_{k,p} qw_{k,j,p,a,s}, \quad \forall j; a \geq 2, 2 \leq s < \eta \tag{26}$$

$$z_{j,a,s} = z_{j,a,\xi(s)} - \sum_{k,p} qw_{k,j,p,a,s}, \quad \forall j; a \geq 2, s \geq \eta \tag{27}$$

$$w_k \in \{0, 1\}, \quad \forall k \tag{28}$$

$$xs_{o,l,s} \in \{0, 1\}, \quad \forall o, l, s \tag{29}$$

$$xw_{p,l,s} \in \{0, 1\}, \quad \forall p, l, s \tag{30}$$

$$ys_{i,o,l,s}, yw_{k,p,l,s}, y_{k,j,p,s}, qs_{i,k,o,a,s}, qw_{k,j,p,a,s}, v_{k,a,s}, h_{k,a,s}, z_{j,a,s} \in Z^+, \quad \forall i, j, k, o, p, a, s \tag{31}$$

This multi-stage stochastic programming model is NP-hard. This follows the fact that the proliferation of variables and constraints to represent conditions and actions in several possible future scenarios. Moreover, the proposed model contains complex structure information of the scenario trees (e.g., inter-stage dependence).

## 4. Solution methodology

Rockafellar and Wets (1991) proposed the progressive hedging algorithm (PHA), which is widely employed to solve stochastic programming problems. The basic idea of PHA is to decompose the stochastic programming problem based on the scenario and iteratively solve penalized versions of the sub-problems to gradually enforce convergence. In each iteration of PHA, an aggregated solution that satisfies the non-anticipativity constraints is formed and penalties are applied in the next iteration based on deviations from that solution (Gade et al., 2016). The individual scenario problems can be easily solved separately. A flowchart of the PHA is illustrated in Fig. 3.

To apply PHA, scenarios are redefined. Fig. 4 presents a sample scenario tree. The standard form and disaggregated form of the scenario tree are shown in Fig. 4($a$) and ($b$), respectively. The paths from the root scenario to the scenarios can be used to describe realizations of the stochastic process from the present stage to the stages where scenarios appear. A full evolution of the stochastic process, a path from the root scenario to the scenarios which belong to the last stage, is called a scenario sequence, indexed by $n \in N$. As shown in Fig. 4($b$), the set of scenario sequences $N$ comprises 6 elements.

To ensure that solutions of the disaggregated form are feasible in the standard form, non-anticipativity constraints must be satisfied. We introduce a set of scenario bundles, indexed by $b(n, t) \in B$. There is a unique corresponding scenario bundle $b(n, t)$ at stage $t$ in scenario sequence $n$. If two or more scenario sequences share the same realization at a stage, this indicates that they share the same scenario bundle. Therefore, the non-anticipativity constraints force the decisions made at this stage to be the same across corresponding scenario sequences. For instance, as shown in Fig. 4($b$), scenario sequences 1 and 2 share scenario bundle 2 at stage 2, i.e., $b(1, 2) = b(2, 2) = 2$. The set of scenario bundles $B$ comprises 4 elements.

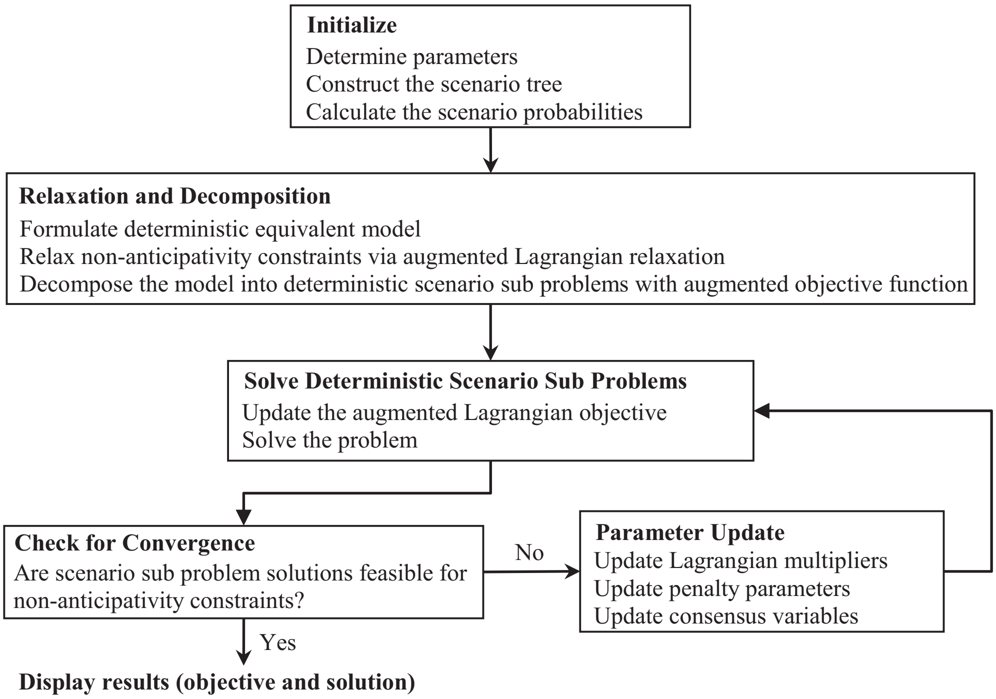


**Fig. 3.** Flowchart of the PHA (Aydin, 2012).

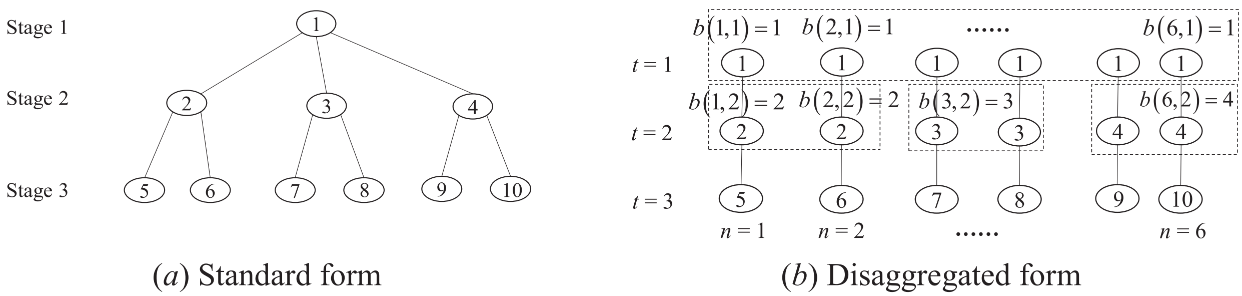


**Fig. 4.** Standard form and disaggregated form of the scenario tree.

More specifically, if a decision is made under scenario sequence 1 at stage 2, then the same decision has to be made under scenario sequence 2 at stage 2. A more specific instance is given after the following definitions.

The additional notations and revised decision variables are presented as follows.

**Additional sets and indices**

| | |
|---|---|
| $N$ | Set of scenario sequences, indexed by $n$. |
| $T$ | Set of the stage, indexed by $t$. |
| $B$ | Set of scenario bundles, indexed by $b(n,t)$. |

**Revised and additional parameters**

| | |
|---|---|
| $\varphi_{k,j,t}$ | Capability of the road from warehouse $k$ to affected location $j$ at stage $t$. |
| $\Pr_n$ | Probability of occurrence of scenario sequence $n$. |
| $CRS_{o,l,t}$ | Unit rental for hiring type $o$ vehicles in quantity interval $l$ at stage $t$. |
| $CRW_{p,l,t}$ | Unit rental for hiring type $p$ vehicles in quantity interval $l$ at stage $t$. |
| $\hat{w}_k$ | Probability of warehouse $k$ being selected to temporarily store supplies. |
| $\widehat{ys}_{i,o,l}^{b(n,t)}$ | Number of type $o$ vehicles used by supplier $i$ in quantity interval $l$ in bundle $b(n,t)$. |
| $\widehat{qw}_{k,j,p,a}^{b(n,t)}$ | Quantity of type $a$ items delivered from warehouse $k$ to affected location $j$ via type $p$ vehicles in bundle $b(n,t)$. |

**Revised decision variables**

| | |
|---|---|
| $w_k^n$ | If warehouse $k$ is selected for temporarily storing supplies, $w_k^n = 1$; otherwise, $w_k^n = 0$. |
| $xs_{o,l,t}^n$ | If type $o$ vehicles are set to the quantity interval $l$ at stage $t$, $xs_{o,l,t}^n = 1$; otherwise, $xs_{o,l,t}^n = 0$. |

| | |
|---|---|
| $xw^n_{p,l,t}$ | If type $p$ vehicles are set to the quantity interval $l$ at stage $t$, $xw^n_{p,l,t}=1$; otherwise, $xw^n_{p,l,t}=0$. |
| $ys^n_{i,o,l,t}$ | Number of type $o$ vehicles used by supplier $i$ in quantity interval $l$ at stage $t$. |
| $yw^n_{k,p,l,t}$ | Number of type $p$ vehicles used by warehouse $k$ in quantity interval $l$ at stage $t$. |
| $y^n_{k,j,p,t}$ | Number of type $p$ vehicles used for delivering supplies from warehouse $k$ to affected location $j$ at stage $t$. |
| $qs^n_{i,k,o,a,t}$ | Quantity of type $a$ items delivered from supplier $i$ to warehouse $k$ via type $o$ vehicles at stage $t$. |
| $qw^n_{k,j,p,a,t}$ | Quantity of type $a$ items delivered from warehouse $k$ to affected location $j$ via type $p$ vehicles at stage $t$. |
| $v^n_{k,a,t}$ | Quantity of type $a$ items at warehouse $k$ at stage $t$. |
| $h^n_{k,a,t}$ | Additional quantity of type $a$ items which need be handled by additional workforce at warehouse $k$ at stage $t$. |
| $z^n_{j,a,t}$ | Shortage of types $a$ items at affected location $j$ at stage $t$. |

In the disaggregated form, parameter network capacity $\phi$ is deterministic for scenario sequence $n$. The multi-stage stochastic model is thus transferred to a multi-stage deterministic model, which is easily solved. Note that a branch and bound algorithm is employed to solve the multi-stage deterministic model, which is performed in CPLEX. For each scenario sequence $n$, the multi-stage deterministic model is presented in Appendix A. We use decision variable $ys$ as an example to show how decisions are synchronized by using the non-anticipativity constraint. For instance, under scenario sequence 1 and 2, the number of vehicles used at stage 2 is $ys^1_{i,o,l,2}, ys^2_{i,o,l,2}, \quad \forall i,o,l$. Because they share scenario bundle 2, to ensure that solutions of the disaggregated form are feasible in the standard form, the non-anticipativity constraint $ys^1_{i,o,l,2}=ys^2_{i,o,l,2}, \quad \forall i,o,l$ must be satisfied.

To facilitate the generation of scenario sub-problem, the consensus variables $\hat{w}_k$, $\widehat{ys}^{b(n,t)}_{i,o,l}$, $\widehat{qw}^{b(n,t)}_{k,j,p,a}$ are defined. The multi-stage deterministic model must consider non-anticipativity constraints, as shown in Eqs. (32)–(34).

$$w^n_k=\hat{w}_k, \quad \forall k,n \tag{32}$$

$$ys^n_{i,o,l,t}=\widehat{ys}^{b(n,t)}_{i,o,l}, \quad \forall i,o,l,t,n \tag{33}$$

$$qw^n_{k,j,p,a,t}=\widehat{qw}^{b(n,t)}_{k,j,p,a}, \quad \forall k,j,p,a,t,n \tag{34}$$

An augmented Lagrangian relaxation technique is usually applied to relax the non-anticipativity constraints, which are moved into the objective function as penalty terms with Lagrangian multipliers and the penalty parameter. $\lambda^w$,$\lambda^{ys}$,$\lambda^{qw}$denote the Lagrangian multipliers, and $\rho^w$,$\rho^{ys}$,$\rho^{qw}$denote the penalty parameters. PHA incrementally enforces non-anticipativity by penalizing deviations from the average values of decision variables (Veliz et al., 2015). Hence, the new objective function of each scenario sub-problem is rewritten as Eq. (35).

$$\begin{aligned} f=\min\sum_t(rc_t+tc_t+hc_t+pc_t)+\sum_k\lambda^w_{k,n}\cdot\left(w^n_k-\hat{w}_k\right)+\frac{1}{2}\rho^w\sum_k\left(w^n_k-\hat{w}_k\right)^2 \\ +\sum_{i,o,l,t}\lambda^{ys}_{i,o,l,t,n}\cdot\left(ys^n_{i,o,l,t}-\widehat{ys}^{b(n,t)}_{i,o,l}\right)+\sum_{k,j,p,a,t}\lambda^{qw}_{k,j,p,a,t,n}\cdot\left(qw^n_{k,j,p,a,t}-\widehat{qw}^{b(n,t)}_{k,j,p,a}\right) \\ +\frac{1}{2}\rho^{ys}\sum_{i,o,l,t}\left(ys^n_{i,o,l,t}-\widehat{ys}^{b(n,t)}_{i,o,l}\right)^2+\frac{1}{2}\rho^{qw}\sum_{k,j,p,a,t}\left(qw^n_{k,j,p,a,t}-\widehat{qw}^{b(n,t)}_{k,j,p,a}\right)^2 \end{aligned} \tag{35}$$

Since quadratic mixed-integer program models are difficult to solve, quadratic penalty components are thus linearized as follows. For the quadratic penalty component which contains the binary decision variable, its linearization form is shown as the second and third components in Eq. (36). Moreover, the method presented by Long et al. (2012) is employed to linearize the quadratic penalty component which contains the integer decision variable. According to their method, $ys$ and $qw$ can be forced to converge by augmenting the objective function, which is expressed as the fourth and fifth components in Eq. (36). Then, Eqs. (37)–(39) are introduced to convert the absolute forms to linearization forms. Hence, the objective function is rewritten as Eq. (40).

$$\begin{aligned} f=\min\sum_t(rc_t+tc_t+hc_t+pc_t)+\sum_k\lambda^w_{k,n}\cdot\left(w^n_k-\hat{w}_k\right)+\frac{1}{2}\rho^w\sum_k\left(w^n_k-2\cdot w^n_k\cdot\hat{w}_k+\hat{w}_k\right) \\ +\sum_{i,o,l,t}\lambda^{ys}_{i,o,l,t,n}\cdot\left|ys^n_{i,o,l,t}-\widehat{ys}^{b(n,t)}_{i,o,l}\right|+\sum_{k,j,p,a,t}\lambda^{qw}_{k,j,p,a,t,n}\cdot\left|qw^n_{k,j,p,a,t}-\widehat{qw}^{b(n,t)}_{k,j,p,a}\right| \end{aligned} \tag{36}$$

$$os_{i,o,l,t}-os'_{i,o,l,t}=ys^n_{i,o,l,t}-\widehat{ys}^{b(n,t)}_{i,o,l}, \forall i,o,l,t,n \tag{37}$$

$$ow_{k,j,p,a,t}-ow'_{k,j,p,a,t}=qw^n_{k,j,p,a,t}-\widehat{qw}^{b(n,t)}_{k,j,p,a}, \forall k,j,p,a,t,n \tag{38}$$

$$os_{i,o,l,t}, os'_{i,o,l,t}, ow_{k,j,p,a,t}, ow'_{k,j,p,a,t}\geq 0, \quad \forall i,j,k,o,p,l,a,t \tag{39}$$

$$f = \min \sum_t (rc_t + tc_t + hc_t + pc_t) + \sum_k \lambda^w_{k,n} \cdot \left( w^n_k - \hat{w}_k \right) + \frac{1}{2}\rho^w \sum_k \left( w^n_k - 2 \cdot w^n_k \cdot \hat{w}_k + \hat{w}_k \right)$$
$$+ \sum_{i,o,l,t} \lambda^{ys}_{i,o,l,t,n} \cdot \left( os_{i,o,l,t} + os'_{i,o,l,t} \right) + \sum_{k,j,p,a,t} \lambda^{qw}_{k,j,p,a,t,n} \cdot \left( ow_{k,j,p,a,t} + ow'_{k,j,p,a,t} \right) \tag{40}$$

The values of consensus variables are usually set as the expected values of the optimal solutions of each scenario-based sub-problem. The value of consensus variable $\hat{w}_k$ is calculated as Eq. (41). A proximal point method (Rockafellar, 1976; Gul et al., 2015) is employed to estimate the values of $\widehat{ys}^{b(n,t)}_{i,o,l}$ by weighted sum calculation, as shown in Eq. (42). The subset of scenario sequences $N'_{b(n,t)}$is introduced, where $N'_{b(n,t)} \subset N$. For instance, as shown in Fig. 4(b), if $b(n,\,2){=}2{,}n \in N'{=}\{1,2\}$, $\widehat{qw}$can be calculated to be the same as $\widehat{ys}$.

$$\hat{w}_k = \sum_n \mathrm{Pr}_n \cdot w^n_k, \forall k \tag{41}$$

$$\widehat{ys}^{b(n,t)}_{i,o,l} = \sum_{n \in N'_{b(n,t)}} \frac{\mathrm{Pr}_n}{\sum_{n \in N'_{b(n,t)}} \mathrm{Pr}_n} ys^n_{i,o,l,t}, \forall i, o, l, t, b(n,t) \tag{42}$$

The method for updating penalty parameters, Lagrangian multipliers, and PHA termination criterion are described below. The penalty parameter has a large impact on the performance of PHA. The algorithm will quickly show convergence and obtain a suboptimal solution if $\rho$ is larger. However, it would converge slowly, and arrive at an approximately optimal solution if $\rho$ is smaller. A method presented by Hvattum and Lokketangen (2009) of updating the penalty parameter based on the information about the convergence rate has a significant impact on obtaining a better solution and the speed of convergence. The implementation of this method is shown in Eqs. (43)–(45), where $\beta^D$ and $\beta^P$ denote the penalty update multipliers. $\rho^w$ and $\rho^{qw}$can be calculated to be the same as $\rho^{ys}$.

For Lagrangian multiplies $\lambda^w$, we update their values by Eqs. (46) and (47). The values of the Lagrangian multipliers $\lambda^w$ are increased (decreased) if $w^{n(r)}_k > \hat{w}^{r-1}_k$ ($w^{n(r)}_k < \hat{w}^{r-1}_k$), which give an incentive to close (open) a warehouse. According to Long et al. (2012), the values of the Lagrangian multipliers $\lambda^{ys}$ are updated in Eqs. (48) and (49). $\lambda^{qw}$ can be calculated to be the same as $\lambda^{ys}$. The termination criteria are set as the gap between optimal solutions of sub-problems and consensus variables being sufficiently small, as shown in Eqs. (50)–(52).

$$\theta^{P(r)} = \sum_{i,o,l,t,n} \left( \widehat{ys}^{b(n,t)(r)}_{i,o,l} - \widehat{ys}^{b(n,t)(r-1)}_{i,o,l} \right)^2 \tag{43}$$

$$\theta^{D(r)} = \sum_{i,o,l,t,n} \left( ys^{n(r)}_{i,o,l,t} - \widehat{ys}^{b(n,t)(r)}_{i,o,l} \right)^2 \tag{44}$$

$$\rho^{ys(r)} = \begin{cases} \beta^D \cdot \rho^{ys(r-1)}, \mathit{if}\ \theta^{D(r-1)} - \theta^{D(r-2)} > 0 \\ \dfrac{1}{\beta^P} \cdot \rho^{ys(r-1)}, \mathit{elseif}\ \theta^{P(r-1)} - \theta^{P(r-2)} > 0 \\ \rho^{ys(r-1)}, \mathit{else} \end{cases} \tag{45}$$

$$\lambda^{w(0)}_{k,n} = \rho^{w(0)} \cdot \left( w^{n(0)}_k - \hat{w}^{(0)}_k \right), \forall k, n \tag{46}$$

$$\lambda^{w(r)}_{k,n} = \lambda^{w(r-1)}_{k,n} + \rho^{w(r-1)} \cdot \left( w^{n(r)}_k - \hat{w}^{(r-1)}_k \right), \forall k, n \tag{47}$$

$$\lambda^{ys(0)}_{i,o,l,t,n} = \rho^{ys(0)} \cdot \left| ys^{n(0)}_{i,o,l,t} - \widehat{ys}^{b(n,t)(0)}_{i,o,l} \right|, \forall i, o, l, t, n \tag{48}$$

$$\lambda^{ys(r)}_{i,o,l,t,n} = \lambda^{ys(r-1)}_{i,o,l,t,n} + \rho^{ys(r-1)} \cdot \left| ys^{n(r)}_{i,o,l,t} - \widehat{ys}^{b(n,t)(r-1)}_{i,o,l} \right|, \forall i, o, l, t, n \tag{49}$$

$$\sum_n \mathrm{Pr}_n \left( \sum_k \left| w^{n(r)}_k - \hat{w}^{(r)}_k \right| \right) \le \varepsilon \tag{50}$$

$$\sum_n \mathrm{Pr}_n \left( \sum_{i,o,l,t} \left| ys^{n(r)}_{i,o,l,t} - \widehat{ys}^{b(n,t)(r)}_{i,o,l} \right| \right) \le \varepsilon \tag{51}$$

$$\sum_n \mathrm{Pr}_n \left( \sum_{k,j,p,a,t} \left| qw^n_{k,j,p,a,t} - \widehat{qw}^{b(n,t)}_{k,j,p,a} \right| \right) \le \varepsilon \tag{52}$$

Although PHA may eventually force an agreement between the decision variables and the consensus variables, many iterations are required (which is very time-consuming) for complex stochastic integer programming problems, such as the presence of binary variables. Variable fixing is a common heuristic strategy (Watson and Woodruff, 2011; Veliz et al., 2015) which is used in PHA for obtaining solutions more quickly. For each PH iteration, a high value of $\hat{w}_k$ indicates that warehouse $k$ would be selected under most scenario sub-problems. We thus present the strategy, i.e. when $\hat{w}_k$ is continuously higher than a given threshold $\sigma$ for several PH iterations to quickly select warehouses.

The implementation of PHA is explained below.

| Step | Description |
|---|---|
| 1 | Set algorithm terminates = false, $r = 0$, $\rho^{w(0)} = 1$, $\rho^{ys(0)} = \rho^{qw(0)} = 0.01$, $\varepsilon = 0.001$; |
| 2 | **While** algorithm terminates = false |
| 3 | **for** $n \in N$ |
| 4 | Solve multi-stage deterministic model (see Appendix A), and collect optimal solutions $w^n_k$, $ys^n_{i,o,l,t}$, $qw^n_{k,j,p,a,t}$ |
| 5 | **end for** |
| 6 | Calculate the values of consensus variables $\hat{w}_k$, $\widehat{ys}^{b(n,t)}_{i,o,l}$ and $\widehat{qw}^{b(n,t)}_{k,j,p,a}$ using Eqs. (41) and (42) |
| 7 | Calculate the initial values of the Lagrangian multipliers $\lambda^{w(0)}_{k,n}$, $\lambda^{ys(0)}_{i,o,l,t,n}$, and $\lambda^{qw(0)}_{k,j,p,a,t,n}$ using Eqs. (46) and (48) |
| 8 | Set $r = 1$ |
| 9 | **for** $n \in N$ |
| 10 | Solve multi-stage deterministic model using new objective Eq. (40), and collect optimal solutions $w^n_k$, $ys^n_{i,o,l,t}$ and $qw^n_{k,j,p,a,t}$ |
| 11 | **end for** |
| 12 | Update the values of consensus variables $\hat{w}_k$, $\widehat{ys}^{b(n,t)}_{i,o,l}$ and $\widehat{qw}^{b(n,t)}_{k,j,p,a}$ using Eqs. (41) and (42) |
| 13 | Update values of the penalty parameters $\rho^{w(r)}$, $\rho^{ys(r)}$ and $\rho^{qw(r)}$ using Eqs. (43)–(45) |
| 14 | Update values of the Lagrangian multipliers $\lambda^{w(r)}_{k,n}$, $\lambda^{ys(r)}_{i,o,l,t,n}$ and $\lambda^{qw(r)}_{k,j,p,a,t,n}$ using Eqs. (47) and (49) |
| 15 | **if** Eqs. (50)–(52) are true |
| 16 | Set algorithm terminates = true |
| 17 | **end if** |
| 18 | **else** |
| 19 | Set $r = r + 1$ |
| 20 | **end else** |
| 21 | **end while** |

## 5. Case study

### 5.1. Estimation of input data

An earthquake took place on April 20, 2013, in Yaan of Sichuan Province, China. This earthquake affected 1.52 million people, including 196 deaths, 11,470 injuries, 21 people missing, and reconstruction budgets exceeding 86 billion Yuan (http://www.csi.ac.cn/). The most serious damage happened in Longmen village, where 99% of the houses collapsed. In this study, the real-world case of an earthquake in Yaan is used to estimate input data. Fig. 6 shows the research region. Below, the sets are configured and the input data for the parameters are estimated.

As defined in the previously mentioned model, eight sets are provided. The set of suppliers ($I$) consists of 10 elements, labeled as 1 through 10. The set of demand locations ($J$) consists of 9 elements. In this study, only the locations in the vicinity of the highway can be selected as temporary warehouse locations. The set of warehouses ($K$) contains 6 elements, labeled 1, 2, 6, 7, 8, and 9. Airplanes and trains are used to transport supplies from suppliers to temporary warehouses during long distance transportation. There are 2 elements in the set of $O$, where 1 and 2 represent airplanes and trains, respectively. Truck and helicopter are used to deliver supplies for local distribution. There are 2 elements in the set of $P$, where 1 and 2 represent trucks and helicopters, respectively. Here, we only consider two temporary warehouses and three types of supplies. The demand of one person for type 1 items and type 3 items stands as a unit, while the demand of four people for type 2 items stands as a unit. The demand for type 3 items is assumed to be 1/8 of the affected population. The earthquake-affected locations, population, and demand for supplies are shown in Table 2.

**Table 2**
Earthquake-affected locations, population, and demand for supplies.

| No. | Location | Affected population | Type 1 | Type 2 | Type 3 |
|---|---|---|---|---|---|
| 1 | Yucheng | 192,000 | 192,000 | 48,000 | 24,000 |
| 2 | Mingshan | 155,100 | 155,100 | 38,775 | 19,387 |
| 3 | Tianquan | 138,000 | 138,000 | 34,500 | 17,250 |
| 4 | Lushan | 120,000 | 120,000 | 30,000 | 15,000 |
| 5 | Baoxing | 54,000 | 54,000 | 13,500 | 6,750 |
| 6 | Qionglai | 367,800 | 367,800 | 91,950 | 45,975 |
| 7 | Dayi | 300,000 | 300,000 | 75,000 | 37,500 |
| 8 | Danling | 99,000 | 99,000 | 24,750 | 12,375 |
| 9 | Pujiang | 156,000 | 156,000 | 39,000 | 19,500 |

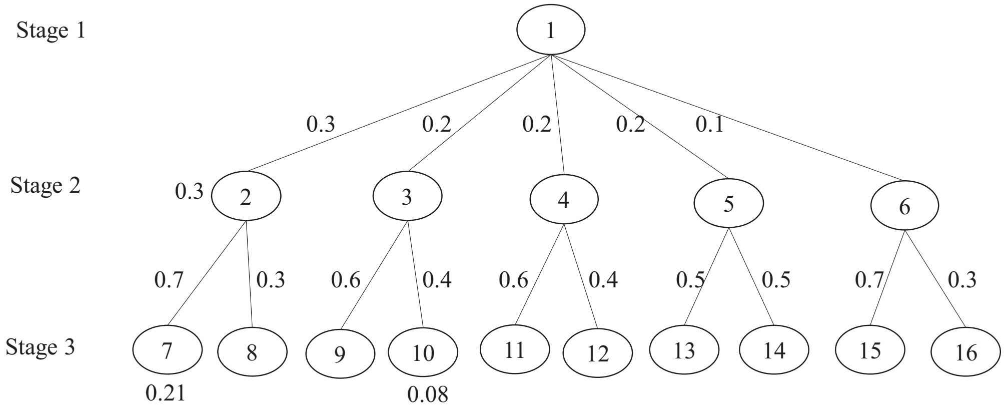


**Fig. 5.** Scenario tree with probabilities for numerical analysis.

**Table 3**
The quantity of airplanes, trains, trucks, and helicopters for each quantity interval.

| Quantity interval | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Airplane | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 |
| Train | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| Truck | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 |
| Helicopter | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 |

Fig. 5 displays the scenario tree for numerical analysis. The set of scenarios ($S$) consists of 16 elements. The set of parent scenarios ($\Omega$) consists of 6 elements. We define that there are four types of secondary disasters: minor, moderate, major, and catastrophe (Moreno et al., 2016). We believe that secondary disasters frequently occur on the first day after an earthquake. We define scenarios 2–6 as the catastrophe, major disaster, moderate disaster, minor disaster, and normal situation, respectively. Scenarios 7, 9, 11, 13, and 15 represent moderate disasters. The remaining scenarios represent normal situations.

To determine the probability of occurrence of each arc, we analyze historical data of the types and frequencies of secondary disasters after major earthquakes in China. Because there is a high probability that major secondary disasters occur on the first day following an earthquake, we assume that the probability of occurrence of the arc to scenario 2 is the greatest, while the arc to scenario 6 is the smallest. Further, the probabilities of the arc to the secondary disaster scenarios 7, 9, 11, 13, and 15 are larger than normal scenarios 8, 10, 12, 14, and 16. As defined in Section 3.1, for instance, the probability of scenario 7 equals the product of 0.3 and 0.7. The network capacity of each stage from the warehouses to affected locations is assumed to be proportional to the demand in the affected locations. It is generated based on a discrete uniform distribution that considers the impacts of secondary disasters. Three rules are considered: 1) based on the real situations in Yaan earthquake, the capacity of routes to affected locations 3, 4, and 5 is 0; 2) for the same stage, the network capacity of serious scenarios is smaller than that of unserious scenarios. For instance, the network capacity of scenario 3 (major disaster) is smaller than that of scenario 5 (minor disaster); 3) the network capacity under the current scenario is larger than or equal to that of its corresponding parent scenario. For instance, the network capacity of scenario 2 is smaller than that of scenario 8. Road network capacity under scenarios and its probabilities can be found in Mendeley data.

In order to assure relief supplies can be transported to disaster-affected locations on time, a certain number of airplanes, trains, trucks, and in some cases even helicopters, must be rented from commercial carriers. Table 3 presents the quantity of airplanes, trains, trucks, and helicopters that can be rented, in quantity intervals. Rental for airplanes, trains, trucks, and helicopters for different quantity intervals and scenarios are shown in Table 4. Rental is determined according to the following rules. For a stage, the rental increases when a large quantity interval is hired. However, for a quantity interval, the rental is higher in the earlier time because of urgency, since as time goes on, the unit rental decreases. Using the airplane as an example, the cost is ¥6,450 (¥ denotes Chinese Yuan) instead of ¥5,750 per airplane if the quantity interval of 6 is hired instead of the quantity interval of 5 under scenario 1. For quantity interval 5, the rental is ¥5,750, ¥5,450 and ¥5,150 per airplane under scenarios 1, 2–6, and 7–16, respectively.

One truck can transport up to 2,500 units and one plane can transport up to 5,000 units (Rottkemper et al., 2012), while a helicopter can move up to 500 units. In reality, most of the capacity of trains are used to transport fuel. We thus assume that one train can transport up to 20,000 units of the studied items. The distance between suppliers and warehouses and the distance between warehouses and disaster-affected locations is presented in Appendix B. All remaining parameters defined in the previously mentioned model are shown in Table 5.

**Table 4**
Rental for airplanes, trains, trucks, and helicopters for different quantity intervals and scenarios.

| Scenario | Vehicle | Quantity interval | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | Airplane | 5,000 | 5,000 | 5,000 | 5,250 | 5,750 | 6,450 | 7,450 | 8,750 | 10,350 | 12,250 |
| | Train | 5,000 | 5,000 | 5,000 | 5,250 | 5,750 | 6,450 | 7,450 | 8,750 | 10,350 | 12,250 |
| 2–6 | Airplane | 4,700 | 4,700 | 4,700 | 4,950 | 5,450 | 6,150 | 7,150 | 8,450 | 10,050 | 11,950 |
| | Train | 4,700 | 4,700 | 4,700 | 4,950 | 5,450 | 6,150 | 7,150 | 8,450 | 10,050 | 11,950 |
| 7–16 | Airplane | 4,400 | 4,400 | 4,400 | 4,650 | 5,150 | 5,850 | 6,850 | 8,150 | 9,750 | 11,650 |
| | Train | 4,400 | 4,400 | 4,400 | 4,650 | 5,150 | 5,850 | 6,850 | 8,150 | 9,750 | 11,650 |
| 1 | Truck | 500 | 500 | 500 | 525 | 575 | 645 | 745 | 875 | 1,035 | 1,225 |
| | Helicopter | 2,500 | 2,500 | 2,500 | 2,625 | 2,875 | 3,225 | 3,725 | 4,375 | 5,175 | 6,125 |
| 2–6 | Truck | 470 | 470 | 470 | 495 | 545 | 615 | 715 | 845 | 1,005 | 1,195 |
| | Helicopter | 2,350 | 2,350 | 2,350 | 2,475 | 2,725 | 3,075 | 3,575 | 4,225 | 5,025 | 5,975 |
| 7–16 | Truck | 440 | 440 | 440 | 465 | 515 | 585 | 685 | 815 | 975 | 1,165 |
| | Helicopter | 2,200 | 2,200 | 2,200 | 2,325 | 2,575 | 2,925 | 3,425 | 4,075 | 4,875 | 5,825 |

**Table 5**
Estimated parameters.

| No | Category | Estimation |
|---|---|---|
| 1 | The handling capacity of a warehouse, the initial quantity of supplies at the warehouse, Quantity of supplies at the supplier | $Q_{:,a} = \{40\ 10\ 5\} \times 10^4 unit$<br>$R_{:,a} = \{20\ 5\ 2.5\} \times 10^4 unit$<br>$U_{:,a} = \{U(5,\ 10)U(20,\ 40)U(2.5,\ 5)\} \times 10^4 unit$ |
| 2 | The volume of supplies, The transport capacity of vehicles O, The transport capacity of vehicles P | $N_a = \{0.1\ 1\ 0.5\} unit,\ ES_o = \{5,000\ 20,000\} unit$<br>$EW_p = \{2,500\ 500\} unit$ |
| 3 | Unit cost for transport, holding supplies, and penalty for the shortage of supplies | $CTS_{o,a} = \begin{Bmatrix} 0.09\ 0.0225\ 0.0225 \\ 0.02\ 0.005\ 0.005 \end{Bmatrix} yuan/unit \cdot km$<br>$CTW_{p,a} = \begin{Bmatrix} 0.04\ 0.01\ 0.01 \\ 0.2\ 0.05\ 0.05 \end{Bmatrix} yuan/unit \cdot km$<br>$CH_a = \{1\ 5\ 10\} yuan/unit \cdot day$<br>$G_a = \{10\ 2,460\ 5,000\} yuan/unit \cdot day$ |

### 5.2. Numerical analysis

(1) General results

CPLEX Optimization Studio 12.6, a commercial optimization package, is widely employed for formulating and solving diverse optimization problems. The experiments were conducted on an Intel Core i5 PC with the processing power of 3.20 GHz and 8GB memory running under Windows 10. For 16 scenarios, the optimal solution can be found within 117 s. Results and observations are discussed below. Based on the results, the locations of suppliers and warehouses are shown in Fig. 6. Fig. 7 shows dynamic vehicles allocation in local distribution under scenarios 2 and 8. Distance is an important factor, and the model tends to select sources which are closer to the earthquake-affected locations. Moreover, selected warehouses are closer to the two groups of the affected locations. As shown in Fig. 7, the warehouse located at location 1 is primarily responsible for the demand of affected locations 1–5, while the other one located at location 7 is primarily responsible for the demand of affected locations 6–9. The helicopters are all used to deliver relief supplies from warehouse 1 to affected locations 3–5. This is because that their road capacity is zero. Furthermore, as a child scenario of scenario 2, vehicles allocation decision of scenario 8 dependent on remaining demand and road capacity. For instance, as a large proportion of demand is satisfied in scenario 2, the number of vehicles used in scenario 8 decreases. As road capacity between locations 1 and 3, 1 and 4 increases, trucks are used to deliver relief supplies.

Fig. 8 shows the number of airplanes, trains, trucks, and helicopters used in various scenarios. Note that the number of vehicles used is different across scenarios at each stage. This is because the road capacity is different in each scenario. For instance, since scenario 4 has larger road capacity than scenario 3, the model tends to use 2 more airplanes and 5 more trucks to deliver relief supplies. When network capacity becomes larger, the decision should be to rent more airplanes and trucks to deliver relief supplies. This is because airplanes have higher time efficiency than trains, and trucks have a much larger capacity than helicopters. The proposed relief distribution with consideration of the state of road network is better at accommodating transportation infrastructure and victims' needs compared to the traditional transshipment network. This could help to improve the use of network capacity and decrease penalty cost.

Table 6 shows the quantity of supplies transported via airplane, train, truck, and helicopter. As shown in Fig. 8, scenarios 2–6 belong to stage 2, and scenario 7–16 belong to stage 3. Although 11 trains were used in stage 2, no items were transported by trains in stage 2. This is because the assigned trains in stage 2 will arrive on stage 3, as shown in Fig. 8 and Table 6. Type 2 and 3 items have higher unit penalty cost, thus they tend to be transported by airplane at stage 2. All helicopters are used for delivering type 2 and 3 items within stage 2. This result reveals that airplanes and helicopters tend

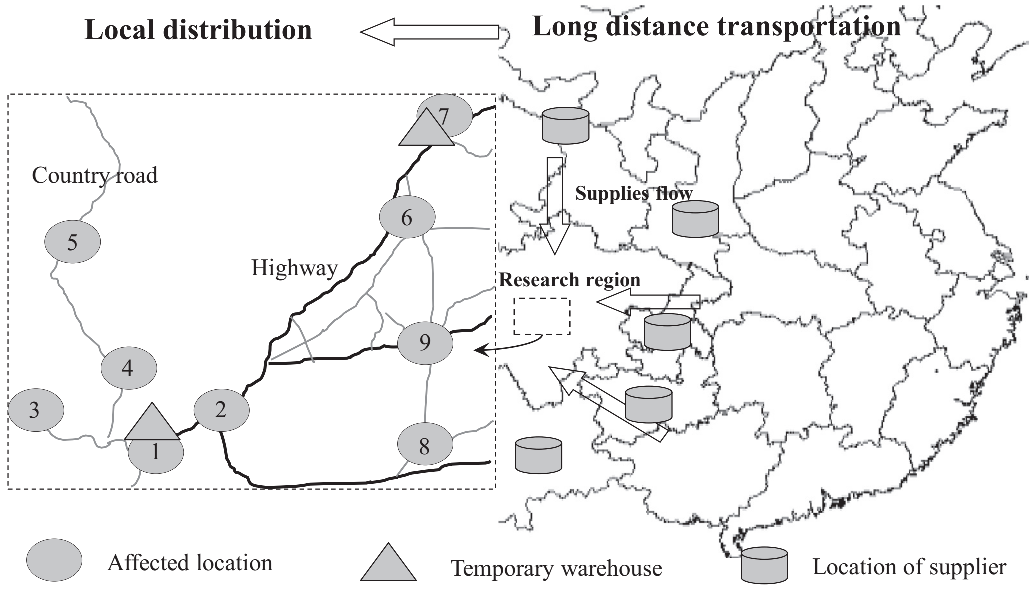


**Fig. 6.** Research region, and locations of suppliers and warehouses.

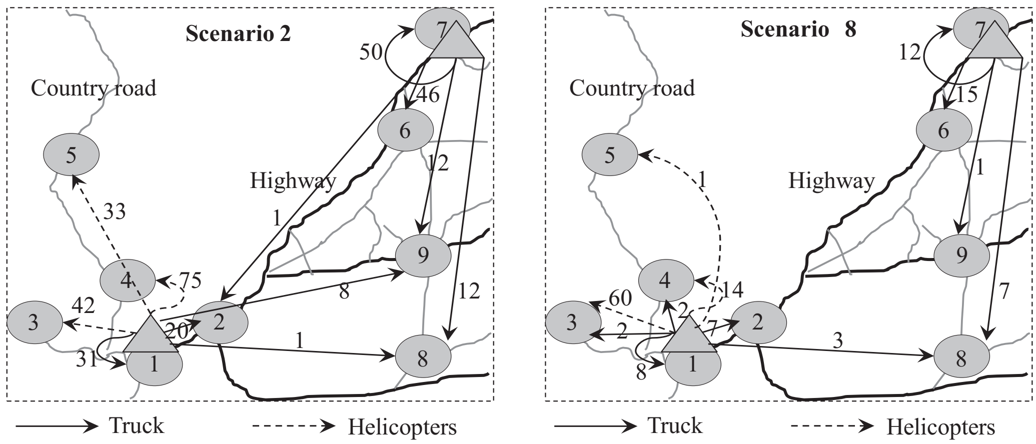


**Fig. 7.** Dynamic vehicles allocation in local distribution.

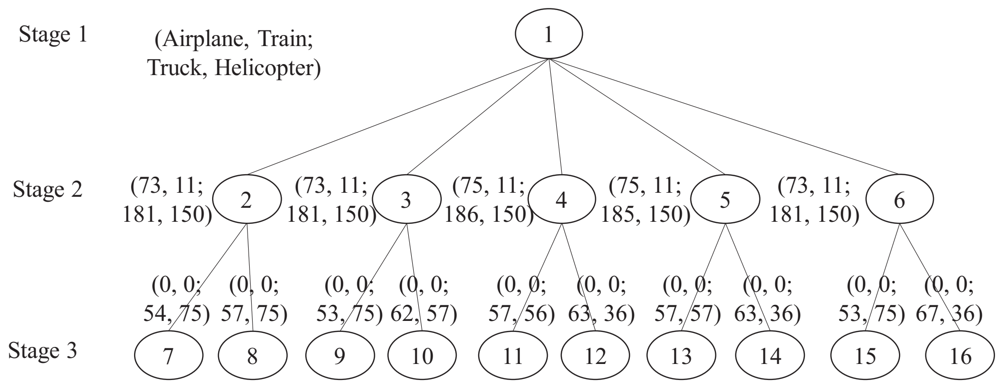


**Fig. 8.** Number of airplanes, trains, trucks, and helicopters used for scenarios.

**Table 6**
Quantity of supplies transported via airplane, train, truck, and helicopter ($\times 10^3$ unit).

| Scenario | Type (via airplane) | | | Type (via train) | | | Type (via truck) | | | Type (via helicopter) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| 2 | 100.0 | 273.0 | 147.7 | 0 | 0 | 0 | 500.0 | 317.5 | 158.7 | 0 | 55.5 | 39.0 |
| 3 | 100.0 | 273.0 | 147.7 | 0 | 0 | 0 | 500.0 | 317.5 | 158.7 | 0 | 55.5 | 39.0 |
| 4 | 100.0 | 283.0 | 147.7 | 0 | 0 | 0 | 500.0 | 327.5 | 158.7 | 0 | 55.5 | 39.0 |
| 5 | 100.0 | 283.0 | 147.7 | 0 | 0 | 0 | 500.0 | 327.5 | 158.7 | 0 | 55.5 | 39.0 |
| 6 | 100.0 | 273.0 | 147.7 | 0 | 0 | 0 | 500.0 | 317.5 | 158.7 | 0 | 55.5 | 39.0 |
| 7 | 0 | 0 | 0 | 1419.0 | 23.0 | 0 | 1269.9 | 0 | 0 | 149.0 | 22.5 | 0 |
| 8 | 0 | 0 | 0 | 1518.7 | 22.5 | 0 | 1369.9 | 0 | 0 | 148.8 | 22.5 | 0 |
| 9 | 0 | 0 | 0 | 1418.7 | 22.5 | 0 | 1269.9 | 0 | 0 | 148.8 | 22.5 | 0 |
| 10 | 0 | 0 | 0 | 1527.9 | 22.5 | 0 | 1469.9 | 0 | 0 | 58.0 | 22.5 | 0 |
| 11 | 0 | 0 | 0 | 1523.7 | 12.5 | 0 | 1369.9 | 0 | 0 | 153.8 | 12.5 | 0 |
| 12 | 0 | 0 | 0 | 1573.7 | 12.5 | 0 | 1519.9 | 0 | 0 | 53.8 | 12.5 | 0 |
| 13 | 0 | 0 | 0 | 1527.9 | 12.5 | 0 | 1369.9 | 0 | 0 | 158.0 | 12.5 | 0 |
| 14 | 0 | 0 | 0 | 1573.7 | 12.5 | 0 | 1519.9 | 0 | 0 | 53.8 | 12.5 | 0 |
| 15 | 0 | 0 | 0 | 1418.7 | 22.5 | 0 | 1269.9 | 0 | 0 | 148.8 | 22.5 | 0 |
| 16 | 0 | 0 | 0 | 1574.2 | 22.5 | 0 | 1574.2 | 5 | 0 | 0 | 17.9 | 0 |

**Table 7**
Quantity of supplies at the warehouses at the beginning of the scenarios ($\times 10^3$ unit).

| Scenario | Type 1 | | Type 2 | | Type 3 | |
|---|---|---|---|---|---|---|
| | Location 1 | Location 7 | Location 1 | Location 7 | Location 1 | Location 7 |
| 2 | 200.0 | 300.0 | 139.8 | 233.2 | 76.4 | 121.3 |
| 3 | 200.0 | 300.0 | 139.8 | 233.2 | 77.4 | 120.3 |
| 4 | 200.0 | 300.0 | 139.8 | 243.2 | 76.4 | 121.3 |
| 5 | 200.0 | 300.0 | 139.8 | 243.2 | 77.4 | 120.3 |
| 6 | 200.0 | 300.0 | 139.8 | 233.2 | 77.4 | 120.3 |
| 7 | 495.9 | 922.8 | 22.5 | 0 | 0 | 0 |
| 8 | 595.8 | 922.8 | 22.5 | 0 | 0 | 0 |
| 9 | 495.9 | 922.8 | 22.5 | 0 | 0 | 0 |
| 10 | 605.1 | 922.8 | 22.5 | 0 | 0 | 0 |
| 11 | 605.1 | 922.8 | 12.5 | 0 | 0 | 0 |
| 12 | 651.3 | 922.8 | 12.5 | 0 | 0 | 0 |
| 13 | 605.1 | 922.8 | 12.5 | 0 | 0 | 0 |
| 14 | 653.5 | 922.8 | 12.5 | 0 | 0 | 0 |
| 15 | 565.8 | 922.8 | 22.5 | 0 | 0 | 0 |
| 16 | 653.5 | 922.8 | 22.5 | 0 | 0 | 0 |

to be used to deliver relief supplies with higher urgency (i.e., type 2 and 3 items), in order to greatly decrease the shortage risk of relief supplies. When type 3 items are all satisfied at stage 2, helicopters are used for delivering type 1 items during stage 3. Since type 1 items have the lowest unit penalty cost, they are transported via train and truck. This is intuitive if demand for types 2 and 3 items is satisfied, and the distribution of regularly consumed supplies becomes routine. It may reduce costs if the distribution for type 1 items is outsourced to logistics companies.

Table 7 shows the quantity of supplies at the warehouse at the beginning of scenarios. The quantity of supplies at the two warehouses at the beginning of stage 3 is zero because the demand for type 3 items is satisfied at the end of stage 2. The demand for type 2 items at affected locations 3, 4 and 5 is not fully supplied at the end of stage 2 due to the limitation of helicopters. The warehouse located at location 1 has to reserve type 2 items at the beginning of stage 3. Type 1 items are mainly transported by trains and delivered at the beginning of stage 3. Therefore, the two warehouses have a large quantity of supplies at this stage.

We next investigate the sensitivity of the costs to the changes in handling capacity, unit penalty costs, inventory, unit transportation cost and unit rental. We modify its value from −50% of the original input to +50%, respectively. A summary of the costs is presented in Table 8. The *rc*, *tc*, *hc* and *pc* represent rental cost, transportation cost, handling cost, and the penalty cost, respectively. Several observations are made based on these results. The optimal solutions of all cases can be found within 500 s.

(2) Effects of handling capacity and unit penalty cost

When handling capacity declines (increases), handling cost tends to increase (decline). This is intuitive, as warehouses can keep their handling capacity stable by employing more (fewer) resources. Moreover, the supplies delivered are barely changed. In practice, the quantity of supplies handled in temporary warehouses usually greatly exceeds their handling capacity. Therefore, selecting warehouses that offer large handling capacity is especially beneficial to maintain a stable flow of relief supplies.

**Table 8**
Sensitivity of costs to the changes in handling capacity, unit penalty costs, inventory, unit transportation cost and unit rental.

| | Handling capacity $Q$ | | | | Unit penalty cost $G$ | | | |
|---|---|---|---|---|---|---|---|---|
| | −50% | −25% | +25% | +50% | −50% | −25% | +25% | +50% |
| *rc* | +1.1 | −0.1 | +0.3 | +4.8 | −11.0 | −8.6 | +10.9 | +10.9 |
| *tc* | −2.5 | −1.4 | +1.4 | +2.0 | −21.7 | −9.3 | +9.6 | +9.7 |
| *hc* | +21.2 | +10.7 | −10.1 | −20.9 | −31.6 | −7.4 | +5.5 | +5.5 |
| *pc* | +1.2 | +0.7 | −0.8 | −0.8 | −39.6 | −20.3 | +18.7 | +42.4 |
| *Total* | +1.6 | +0.8 | −0.8 | −1.4 | −33.3 | −15.9 | +14.9 | +29.5 |
| | Inventory of supplier $U$ | | | | Inventory of warehouse $R$ | | | |
| *rc* | −5.6 | −4.1 | +0.8 | +1.7 | +7.5 | +4.5 | −1.9 | −5.9 |
| *tc* | +25.1 | +5.5 | −5.2 | −7.3 | +17.1 | +8.3 | −8.1 | −16.5 |
| *hc* | −4.9 | −3.8 | +0.8 | +1.0 | +7.4 | +3.6 | −3.4 | −7.1 |
| *pc* | +3.0 | +2.4 | +0.1 | −0.6 | +2.5 | +1.3 | −1.4 | −2.6 |
| *Total* | +8.7 | +2.7 | −1.4 | −2.4 | +7.2 | +3.6 | −3.5 | −7.1 |
| | Unit transport cost via vehicle $o$ $CTS$ | | | | Unit transport cost via vehicle $p$ $CTW$ | | | |
| *rc* | +11.5 | +7.0 | −6.6 | −8.7 | +2.8 | +1.4 | −5.4 | −4.1 |
| *tc* | −38.9 | −17.1 | +15.1 | +33.4 | −2.4 | −1.7 | −1.6 | −0.2 |
| *hc* | +8.0 | +4.1 | −4.0 | −7.4 | +1.4 | +1.1 | −2.5 | −3.4 |
| *pc* | −6.6 | −4.2 | +4.2 | +6.2 | −0.8 | −0.1 | +2.1 | +2.1 |
| *Total* | −14.6 | −7.1 | +6.6 | +12.9 | −1.1 | −0.5 | +0.5 | +0.9 |
| | Unit rental for vehicle $o$ $CRS$ | | | | Unit rental for vehicle $p$ $CRW$ | | | |
| *rc* | −15.0 | −6.5 | +7.9 | +13.9 | −36.0 | −18.0 | +18.2 | +34.5 |
| *tc* | −0.9 | +0.2 | +0.4 | 0.0 | −0.3 | −0.3 | 0.0 | −0.7 |
| *hc* | −0.1 | +0.2 | +0.5 | 0.0 | +1.0 | +0.8 | 0.0 | −0.8 |
| *pc* | +0.6 | −0.1 | −0.2 | 0.0 | +0.1 | +0.1 | 0.0 | +0.5 |
| *Total* | −0.2 | −0.1 | +0.2 | +0.2 | −0.6 | −0.3 | +0.3 | +0.7 |

Total costs tend to decrease with lower unit penalty cost and increase with higher unit penalty cost. The quantity of type 2 and 3 items delivered to the affected locations changes slightly. However, the delivered quantity of type 1 items greatly decreases (increases) with lower (higher) unit penalty cost. Since type 1 items have the lowest unit penalty cost, the model tends to hire fewer vehicles to deliver them. This case shows that distribution decisions for type 1 items are sensitive to the changes in unit penalty cost. In practice, the tolerance of managers to unsatisfied demand can be revealed by unit penalty cost.

(3) Effects of inventory

When suppliers' inventory decreases, its impact on costs is larger than decrease of warehouses' inventory. This is because supplies should be transported from further sources. Therefore, reserving a certain quantity of supplies that are adjacent to areas prone to earthquakes is particularly beneficial. When suppliers' inventory increase, its impact on costs is smaller than increasing of warehouses' inventory. This indicates that the larger the quantity of initial supplies reserved in areas that are prone to earthquakes, the more time- and cost-efficiencies can be achieved in relief distribution operations. In practice, to achieve above-mentioned goals, the relief agency should pre-positioning more relief supplies at strategic warehouses on one hand. On the other hand, they should establish a close relationship with commercial suppliers (e.g., supermarkets). Once needed, these suppliers could deliver relief supplies to designated locations in the first time to ensure that the affected people can receive timely and effective assistance.

(4) Effects of transportation cost

Total costs tend to increase (decrease) with the increase (decrease) of unit transportation cost of airplanes and trains. It has a small influence on costs when changing the unit transportation cost of trucks and helicopters. According to the results, the quantity of type 1 items delivered tends to increase when the unit transportation cost of airplanes and trains decrease. This implies that negotiating for low transportation fees for long distance transportation is conducive to supply of type 1 items.

(5) Effects of rental and quantity of vehicles

Apart from rental cost, another three sub-costs and total costs are slightly affected to changes in the unit rental of vehicles. As rental cost accounts for a small percentage of total costs, increasing unit rental has no significant influence on distribution decisions. This can happen only if the number of vehicles is large enough. Total costs and distribution decisions are very sensitive to changes in the quantity of vehicles, and the results are analyzed below.

Penalty cost increases sharply when the quantity of *VW* (i.e., trucks and helicopters) decrease by 50%, or the quantity of *VS* (i.e., airplanes and trains) decrease by 75%. This is because no enough vehicles could be hired. For instance, in the case

**Table 9**
Sensitivity of the costs to the changes in the quantity of vehicles.

| | Quantity of vehicle *o VS* | | | | | Quantity of vehicle *p VW* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | −75% | −50% | −25% | +25% | +50% | −50% | −25% | +25% | +50% | +75% | +100% |
| *rc* | −23.5 | +30.3 | +8.3 | −2.4 | −2.3 | −13.0 | −4.7 | +5.8 | +0.3 | −9.2 | −16.6 |
| *tc* | −47.6 | 0.0 | +0.2 | 0.0 | 0.0 | −29.2 | −7.3 | +3.6 | +5.4 | +5.4 | +5.4 |
| *hc* | −43.4 | 0.0 | +0.2 | 0.0 | 0.0 | −20.5 | −5.2 | +3.6 | +4.0 | +4.0 | +4.0 |
| *pc* | +712.5 | 0.0 | −0.1 | 0.0 | 0.0 | +331.2 | +81.5 | −70.6 | −80.3 | −80.3 | −80.3 |
| *Total* | +421.5 | +0.5 | +0.1 | 0.0 | 0.0 | +193.8 | +47.7 | −42.1 | −47.6 | −47.8 | −47.9 |

**Table 10**
Sensitivity of the costs to the changes in the quantity of helicopters.

| | −100% | −75% | −50% | −25% | +25% | +50% | +75% | +100% |
|---|---|---|---|---|---|---|---|---|
| *rc* | −69.1 | −40.0 | −12.4 | −5.8 | +6.7 | +2.2 | −6.8 | −14.3 |
| *tc* | −25.7 | −21.0 | −14.1 | −6.4 | +3.5 | +5.4 | +5.4 | +5.4 |
| *hc* | −23.7 | −14.7 | −11.0 | −5.2 | +3.4 | +4.0 | +4.0 | +4.0 |
| *pc* | +1015.8 | +443.1 | +211.4 | +81.1 | −70.5 | −80.3 | −80.3 | −80.3 |
| *Total* | +615.5 | +265.1 | +125.1 | +47.6 | −42.0 | −47.6 | −47.8 | −47.9 |

of there being 150 trucks and 75 helicopters available at most when *VW* decreases by 50%. However, the largest demand for trucks and helicopters is 186 and 150 under scenario 2–6 (see Fig. 8). We also observe that transportation cost and handling cost tend to decrease when *VW* decreases. This is explained by the fact that the model tends to stop transporting relief supplies to warehouses if supplies cannot reach affected locations. Reducing the fleet size leads to a much less efficient distribution of relief supplies. According to the results, penalty cost is particularly sensitive to the changes in the quantity of helicopters. Penalty cost will increase ten folds when the quantity of helicopter decreases to zero. With the increase in the quantity of helicopters, penalty cost decrease until the quantity increases up to 50%. This result reveals that the expansion of fleet size cannot always guarantee a marginal decrease in penalty cost and total costs. The sensitivity of the costs to the changes in the quantity of vehicles is investigated below and shown in Table 9. Table 10 shows the sensitivity of the costs to the changes in the quantity of helicopters.

In summary, these results imply that relief agencies should focus on establishing agreements with carriers that own and operate large vehicle fleets, even though this may lead to higher unit rental. This would be particularly beneficial for improving overall disaster relief distribution. Moreover, infrastructure is often damaged in disasters, network disruption can strongly impact distribution decisions if there is no backup distribution plan, such as helicopter transportation or air-drop of supplies. The primary insight offered by a variety of solutions is that relief agencies can choose a solution that will have good overall effectiveness for disaster relief distribution.

(6) Effects of road capacity and quantity of helicopters

There is no general approach to design road capacity of scenarios. Hence, different road capacity sets may result in various distribution decisions. Relief agencies may want to know the tendency of total costs to changes in road capacity. For road capacity, we modify its value from −50% of the original input to +50%. Furthermore, this is intuitive, increasing quantity of helicopter is an efficient way to reduce the impact of decreasing of road capacity on total costs. Therefore, we will next illustrate the tendency of total costs to changes in road capacity and quantity of helicopters. For the quantity of helicopters, we modify its value from −100% of the original input to +100%. Fig. 9 shows the tendency of total costs to changes in road capacity and quantity of helicopters.

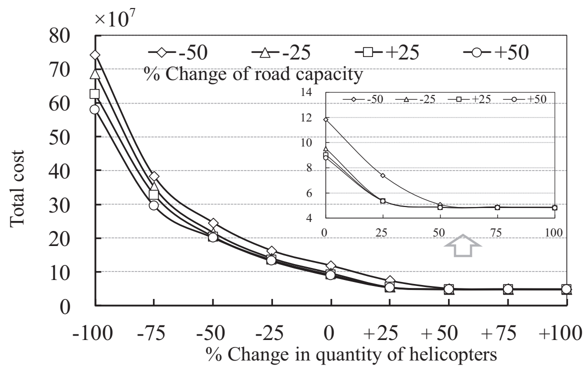


**Fig. 9.** Tendency of total costs to changes in road capacity and quantity of helicopters.

**Table 11**
Performance of PHA to the changes in the penalty update multipliers.

| $qw(\beta^D,\beta^P)$ | $w(\beta^D,\beta^P)$ | Iterations | CPU time (s) | Costs (CNY) | GAP (%) |
|---|---|---|---|---|---|
| (1,1) | (1,1) | 55 | 571 | 92,883,947 | 0.02 |
| (1,1) | (1,2) | 55 | 571 | 92,883,947 | 0.02 |
| (1,1) | (1.5,2) | 46 | 481 | 92,881,839 | 0.02 |
| (1,1) | (2,2) | 102 | 1,041 | 94,027,870 | 1.25 |
| (1,2) | (1,1) | 55 | 571 | 92,883,947 | 0.02 |
| (1,2) | (1,2) | 55 | 571 | 92,883,947 | 0.02 |
| (1,2) | (1.5,2) | 46 | 481 | 92,881,839 | 0.02 |
| (1,2) | (2,2) | 58 | 601 | 92,961,761 | 0.11 |
| (1.5,2) | (1,1) | 78 | 801 | 92,881,593 | 0.02 |
| (1.5,2) | (1,2) | 66 | 681 | 92,875,622 | 0.01 |
| (1.5,2) | (1.5,2) | 41 | 431 | 92,878,985 | 0.02 |
| (1.5,2) | (2,2) | 63 | 651 | 133,362,954 | 43.61 |
| (2,2) | (1,1) | 81 | 831 | 100,098,585 | 7.79 |
| (2,2) | (1,2) | 86 | 881 | 92,877,565 | 0.02 |
| (2,2) | (1.5,2) | 47 | 491 | 92,882,207 | 0.02 |
| (2,2) | (2,2) | 64 | 661 | 95,492,404 | 2.83 |

According to the results, when the quantity of helicopter decrease, total costs tend to increase sharply. For instance, when the quantity of helicopter and road capacity decrease by 100% and 50%, respectively, total costs increase by 527%. When the quantity of helicopter decrease, +50% change in road capacity has the lowest total costs, followed by +25%, −25%, −50%. This occurs because of unsatisfied demand for relief supplies is higher due to lower road capacity. We also observe that the decrease in the quantity of helicopter has a greater impact on total costs than the increase in the quantity of helicopter. For instance, total costs decrease by only 59% (compared to 527%) when the quantity of helicopter increase from 0 to 100% for −50% change of road capacity. As the quantity of helicopter increase, the impact of decreasing of road capacity on total costs would vanish. For instance, for +50%, +25%, −25%, −50% changes of road capacity, their total costs are almost the same when the quantity of helicopter increase up to 50%. This implies that establishing agreements with carriers that offer a large number of helicopters would be particularly beneficial for disaster relief distribution, especially for highly uncertainty of road network capacity.

### *5.3. Performance of PHA*

In this section, we test the solution quality and computational speed of PHA using different datasets. The PHA was coded in MATLAB 2013a environment, and we use CPLEX 12.6 as a solver of each sub-problem. For the 16 scenario cases with set $ys(\beta^D,\beta^P) = (2,2)$, the performance of PHA to the changes in the penalty update multipliers is shown in Table 11. The sub-problems can be solved within 3 s by CPLEX. In particular, the sub-problem can be solved within 1 s after the value of variable w is fixed. Results show that PHA can converge with the least time consumption involved when $qw(\beta^D,\beta^P) = (1.5,2)$. However, when $qw(\beta^D,\beta^P) = (1.5,2)$, $w(\beta^D,\beta^P) = (1, 2)$, the overall costs improve by 0.01% while computational speed falls 58%. We also observed that worse solutions can be obtained when set $w(\beta^D,\beta^P) = (2,2)$.

As shown before, the optimal solutions of the 16 scenarios can be found within 117 s using CPLEX. PHA took 431 s and had a 0.02% gap to the optimal objective value. However, multi-stage stochastic programs become time-consuming when using commercial solvers (such as CPLEX or GUROBI) to solve large problems. We set $ys(\beta^D,\beta^P) = (2,2)$, $qw(\beta^D,\beta^P) = w(\beta^D,\beta^P) = (1.5,2)$, Table 12 shows the performances of CPLEX and PHA for the proposed model with different datasets. The upper and lower bounds are obtained when we stopped CPLEX after running for 3,600 s. The results demonstrate that the PHA can obtain a much better upper boundary within less time for problems with larger datasets of warehouses and scenarios. Moreover, the use of CPLEX for large problems (such as more suppliers, warehouses, affected locations, and scenarios) is not feasible because of limitations in memory and space. For the case of 9 warehouses and 50

**Table 12**
Performances of CPLEX and PHA for the proposed model with different datasets.

| $(K, S)$ | CPLEX | | | PHA | | |
|---|---|---|---|---|---|---|
| | Upper bound ($\times 10^7$) | Lower bound ($\times 10^7$) | GAP (%) | Costs ($\times 10^7$) | CPU time (s) | GAP (%) |
| (6,40) | 9.939 | 9.936 | 0.03 | 9.942 | 916 | 0.06 |
| (6,50) | 10.868 | 10.814 | 0.49 | 10.837 | 772 | 0.21 |
| (6,60) | 158.417 | 10.807 | 93.18 | 10.824 | 1,053 | 0.16 |
| (9,40) | 8.018 | 8.015 | 0.04 | 8.056 | 989 | 0.5 |
| (9,50) | 23.916 | 8.940 | 62.62 | 8.941 | 1,277 | 0.01 |
| (9,60) | 24.271 | 8.942 | 63.16 | 8.957 | 1,543 | 0.17 |

scenarios, CPLEX displays out of memory status after running for 5,685 s. This evaluation demonstrates the solution quality and computational advantages of using PHA.

Before the value of variable $w$ is fixed, for datasets with larger suppliers, warehouses or affected locations, a scenario sub-problem would expend more time. In 9 warehouses cases, a sub-problem expends at least 5 more seconds. PHA works better if employing a more efficient algorithm instead of CPLEX for solving subproblems. The idea of PHA is to decompose the problem, and its advantages gradually emerge as the scenario size grows. However, PHA requires large periods of time to solve scenario sub-problems. This can be dramatically reduced by using the parallelization method.

## 6. Conclusions

A multi-stage stochastic programming model is developed to coordinate vehicles and schedule distribution plans. Supplier capacity, local warehouses handling capacity, multiple transportation modes, and the uncertainty attached to network capacity are integrated into our model to study their effects on distribution decisions. A case study is presented to study the applicability of the proposed model, along with solution quality and the computational advantages associated with PHA.

Based on the results of the numerical analysis, several practical implications are identified in this study. (1) The larger the quantity of initial supplies reserved in areas that are prone to earthquakes, the more time- and cost-efficiencies can be gained by relief distribution operations. (2) The expansion of fleet size should guarantee that increasing a vehicle can bring a large marginal decrease in penalty cost and total cost. (3) Relief agencies should focus on being cost-efficient in long distance transportation by negotiating for low transportation fees. They should focus on the reliability of local distribution by preparing backup distribution plans (such as helicopters or air-drops). The proposed model can be used to study a variety of strategies on location, inventory, and distribution of relief supplies so that relief agencies can make plans for relief distribution when faced with secondary disasters.

Our research suggests several directions for future research. (1) Carrier reliability. This paper implies that the availability of vehicles is not affected. However, there may well be risks involved when carriers do not meet vehicle demands for quantity and timing because of uncertainty. Incorporating carrier reliability into the model may be worth exploring in future research. (2) Investigating carrier selection criteria and types of contracts. This paper does not focus on the details of carrier selection criteria, nor on the exact type of contract with carriers. There is a need for new types of relationships and contracts that allow for more flexible or incentive-based terms, and these relationships should enable parties to equitably share those risks and benefits (Balcik et al., 2010). (3) Considering deprivation costs, which is caused by the lack of access to a good or service (Cantillo et al., 2018). In most disaster contexts, relief supplies are frequently insufficient to satisfy the needs of all affected locations. To allocate scarce resources efficiently, trade-offs between economic costs and deprivation costs should be considered.

## Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (NSFC) under grants 71503185, 71874123, 71841036, 71602114, 71601113; in part by the Science and Technology Commission of Shanghai Municipality under grant 18DZ1206802; and in part by the Ministry of Education of the People's Republic of China under grant 18JZDW02. We also appreciate Prof. Zhe Liang of Tongji University and the two anonymous reviewers for their valuable suggestions which help us a lot in improving the paper.

## Appendix A. The multi-stage deterministic model

For each scenario sequence $n$, the multi-stage deterministic model is formulated as follows.

$$f = \min \sum_{t} (rc_t + tc_t + hc_t + pc_t)$$

$$rc_t = \sum_{i,o,l} \left( ys^n_{i,o,l,t} \cdot CRS_{o,l,t} \right) + \sum_{k,p,l} \left( yw^n_{k,p,l,t} \cdot CRW_{p,l,t} \right), \quad \forall t$$

$$tc_t = \sum_{i,k,o,a} \left( CTS_{o,a} \cdot HS_{i,k} \cdot qs^n_{i,k,o,a,t} \right) + \sum_{k,j,p,a} \left( CTW_{p,a} \cdot HW_{k,j} \cdot qw^n_{k,j,p,a,t} \right), \quad \forall t$$

$$hc_t = \sum_{k,a} CH_a \cdot h^n_{k,a,t}, \quad \forall t$$

$$pc_t = \sum_{j,a} G_a \cdot z^n_{j,a,t}, \quad \forall t$$

$$\sum_{k} w^n_k \leq \gamma$$

$$w_k^n \le \sum_{p,l,t} yw_{k,p,l,t}^n \le M \cdot w_k^n, \quad \forall k$$

$$\sum_l xs_{o,l,t}^n \le 1, \quad \forall o, t$$

$$\sum_l xw_{p,l,t}^n \le 1, \quad \forall p, t$$

$$xs_{o,l,t-1}^n \le \sum_i ys_{i,o,l,t}^n \le xs_{o,l,t-1}^n \cdot VS_{o,l}, \quad \forall o, l; t \ge 2$$

$$xw_{p,l,t-1}^n \le \sum_k yw_{k,p,l,t}^n \le xw_{p,l,t-1}^n \cdot VW_{p,l}, \quad \forall p, l; t \ge 2$$

$$\sum_j y_{k,j,p,t} = \sum_l yw_{k,p,l,t}, \quad \forall k, p; t \ge 2$$

$$qs_{i,k,1,a,1}^n = 0, \quad \forall i, k, a, n$$

$$qs_{i,k,2,a,t}^n = 0, \quad \forall i, k, o, a; t \le 2$$

$$\sum_{k,a} qs_{i,k,1,a,t}^n \cdot \mu_a \le \sum_l ys_{i,1,l,t}^n \cdot ES_1, \quad \forall i; t \ge 2$$

$$\sum_{k,a} qs_{i,k,2,a,t}^n \cdot \mu_a \le \sum_l ys_{i,2,l,t-1}^n \cdot ES_2, \quad \forall i; t \ge 3$$

$$qw_{k,j,p,a,1}^n = 0, \quad \forall k, j, p, a$$

$$\sum_a qw_{k,j,p,a,t}^n \cdot \mu_a \le y_{k,j,p,t} \cdot EW_p, \quad \forall k, j, p; t \ge 2$$

$$\sum_{k,o} qs_{i,k,o,a,t}^n \le U_{i,a}, \quad \forall i, a, t$$

$$\sum_a qw_{k,j,1,a,t}^n \cdot \mu_a \le \varphi_{k,j,t}, \quad \forall k, j, t$$

$$v_{k,a,1}^n = R_{k,a} \cdot w_k^n, \quad \forall k, a$$

$$v_{k,a,t}^n = v_{k,a,t-1}^n + \sum_{i,o} qs_{i,k,o,a,t}^n - \sum_{j,p} qw_{k,j,p,a,t}^n, \quad \forall k, a; t \ge 2$$

$$h_{k,a,t}^n \ge \sum_{i,o} qs_{i,k,o,a,t}^n + \sum_{j,p} qw_{k,j,p,a,t}^n - Q_{k,a}, \quad \forall k, a, t$$

$$z_{j,a,1}^n = 0, \quad \forall j, a$$

$$z_{j,1,t}^n = D_{j,1} - \sum_{k,p} qw_{k,j,p,1,t}^n, \quad \forall j; t \ge 2$$

$$z_{j,a,2}^n = D_{j,a} - \sum_{k,p} qw_{k,j,p,a,2}^n, \quad \forall j; a \ge 2$$

$$z_{j,a,t}^n = z_{j,a,t-1}^n - \sum_{k,p} qw_{k,j,p,a,t}^n, \quad \forall j; a \ge 2, t \ge 3$$

$$w_k^n \in \{0, 1\}, \quad \forall k$$

$$xs_{o,l,t}^n \in \{0, 1\}, \quad \forall o, l, t$$

$$xw_{p,l,t}^n \in \{0, 1\}, \quad \forall p, l, t$$

$$ys_{i,o,l,t}^n, yw_{k,p,l,t}^n, y_{k,j,p,t}^n, qs_{i,k,o,a,t}^n, qw_{k,j,p,a,t}^n, v_{k,a,t}^n, h_{k,a,t}^n, z_{j,a,t}^n \in Z^+, \quad \forall i, j, k, o, p, a, t$$

## Appendix B. Distance matrix

Distance between suppliers and warehouses (unit: km)

| Supplier | Warehouses' location | | | | | |
|---|---|---|---|---|---|---|
| | Yucheng | Mingshan | Qionglai | Dayi | Danling | Pujiang |
| Zhenzhou | 1319 | 1309 | 1257 | 1239 | 1311 | 1279 |
| Xian | 842 | 832 | 781 | 763 | 835 | 803 |
| Lanzhou | 989 | 979 | 1026 | 1008 | 1080 | 1048 |
| Wuhan | 1301 | 1291 | 1229 | 1203 | 1206 | 1225 |
| Changsha | 1329 | 1319 | 1280 | 1262 | 1241 | 1276 |
| Nanchang | 1622 | 1612 | 1573 | 1520 | 1550 | 1569 |
| Nanning | 1284 | 1274 | 1256 | 1266 | 1216 | 1257 |
| Kunming | 766 | 756 | 831 | 853 | 810 | 824 |
| Guiyang | 717 | 707 | 689 | 700 | 650 | 691 |
| Chongqing | 445 | 435 | 397 | 379 | 367 | 393 |

Distance between warehouses and disaster-affected locations (unit: km)

| Location | Yucheng | Mingshan | Tianquan | Lushan | Baoxing | Qionglai | Dayi | Danling | Pujiang |
|---|---|---|---|---|---|---|---|---|---|
| Yucheng | 15 | 13.3 | 41.1 | 35.3 | 77.9 | 67.6 | 90.4 | 84.5 | 60.9 |
| Mingshan | 13.3 | 15 | 51.8 | 45.9 | 88.6 | 57.5 | 80.3 | 74.3 | 50.8 |
| Qionglai | 67.6 | 57.5 | 106.1 | 100.2 | 142.9 | 15 | 27.1 | 55.1 | 32.1 |
| Dayi | 90.4 | 80.3 | 128.9 | 123.1 | 165.7 | 27.1 | 15 | 83.7 | 61.5 |
| Danling | 84.5 | 74.3 | 123 | 117.1 | 159.8 | 55.1 | 83.7 | 15 | 24.1 |
| Pujiang | 60.9 | 50.8 | 99.4 | 93.5 | 136.2 | 32.1 | 61.5 | 24.1 | 15 |

## Mendeley data

Mendeley data for road network capacity associated with this article can be found in the online version.

## References


Ahmadi, M., Seifi, A., Tootooni, B., 2015. A humanitarian logistics model for disaster relief operation considering network failure and standard relief time: a case study on San Francisco district. Transp. Res. Part E 75, 145–163.

Alem, D., Clark, A., Moreno, A., 2016. Stochastic network models for logistics planning in disaster relief. Eur. J. Oper. Res. 255 (1), 187–206.

Aydin, N., 2012. Sampling Based Progressive Hedging Algorithms for Stochastic Programming Problems Ph.D. Thesis, System & Industrial Engineering. Wayne State University, Detroit, MI.

Balcik, B., Beamon, B.M., 2008. Facility location in humanitarian relief. Int. J. Logist. 11 (2), 101–121.

Balcik, B., Beamon, B.M., Krejci, C.C., Muramatsu, K.M., Ramirez, M., 2010. Coordination in humanitarian relief chains: practices, challenges and opportunities. Int. J. Prod. Econ. 126 (1), 22–34.

Barbarosoğlu, G., Özdamar, L., Cevik, A., 2002. An interactive approach for hierarchical analysis of helicopter logistics in disaster relief operations. Eur. J. Oper. Res. 140 (1), 118–133.

Baskaya, S., Ertem, M.A., Duran, S., 2017. Pre-positioning of relief items in humanitarian logistics considering lateral transhipment opportunities. Socioecon. Plann. Sci. 57, 50–60.

Cantillo, V., Serrano, I., Macea, L.F., Holguín-Veras, J., 2018. Discrete choice approach for assessing deprivation cost in humanitarian relief operations. Socioecon. Plann. Sci. 63, 33–46.

China Earthquake Administration. (2008, Aug 29). Distributions and statistics of aftershocks of the M8.0 Wenchuan earthquake (in Chinese). Available at http://www.cea.gov.cn/manage/html/8a8587881632fa5c0116674a018300cf/_content/08_05/30/1212117488687.html. Accessed Oct 12, 2016.

Crainic, T.G., Hewitt, M., Rei, W., 2014. Scenario grouping in a progressive hedging-based meta-heuristic for stochastic network design. Comput. Oper. Res. 43, 90–99.

Dong, J.X., Lee, C.Y., Song, D.P., 2015. Joint service capacity planning and dynamic container routing in shipping network with uncertain demands. Transp. Res. Part B 79, 231–243.

Dong, Z.S., 2015. Efficient Design of Inbound Logistics Networks Ph.D. Thesis, School of Civil & Environmental Engineering. Cornell University, Ithaca, NY.

Dong, Z.S., Turnquist, M.A., 2015. Combining service frequency and vehicle routing for managing supplier shipments. Transp. Res. Part E 75, 145–163.

Dong, Z.S., Hu, S.L., Turnquist, M.A., 2018. Resilient route design for collection of material from suppliers. Transp. Sci. (under review).

Elçi, Ö., Noyan, N., 2018. A chance-constrained two-stage stochastic programming model for humanitarian relief network design. Transp. Res. Part B 108, 55–83.

Ferrer, J.M., Martín-Campo, F.J., Ortuño, M.T., Pedraza-Martínez Alfonso, J., Tirado, G., Vitoriano, B., 2018. Multi-criteria optimization for last mile distribution of disaster relief aid: test cases and applications. Eur. J. Oper. Res. 269 (2), 501–515.

Gade, D., Hackebeil, G., Ryan, S.M., Watson, J.-P., Wets, R.J.-B., Woodruff, D.L., 2016. Obtaining lower bounds from the progressive hedging algorithm for stochastic mixed-integer programs. Math. Program. 157 (1), 47–67.

Gul, S., B. Denton, T., Fowler, J.W., 2015. A progressive hedging approach for surgery planning under uncertainty. INFORMS J. Comput. 27 (4), 755–772.

Guo, G., Hackebeil, G., Ryan, S.M., Watson, J.P., Woodruff, D.L., 2015. Integration of progressive hedging and dual decomposition in stochastic integer programs. Oper. Res. Lett. 43 (3), 311–316.

Hvattum, L.M., Løkketangen, A., 2009. Using scenario trees and progressive hedging for stochastic inventory routing problems. J. Heurist. 15 (6), 527–557.

Kim, G., Wu, K., Huang, E., 2015. Optimal inventory control in a multi-period newsvendor problem with non-stationary demand. Adv. Eng. Inf. 29 (1), 139–145.

Liu, Y., Lei, H., Wu, Z., Zhang, D., 2018. A robust model predictive control approach for post-disaster relief distribution. Comput. Ind. Eng. doi:10.1016/j.cie.2018.09.005.

Listes, O., Dekker, R., 2005. A scenario aggregation–based approach for determining a robust airline fleet composition for dynamic capacity allocation. Transp. Sci. 39 (3), 367–382.

Long, Y., Lee, L.H., Chew, E.P., 2012. The sample average approximation method for empty container repositioning with uncertainties. Eur. J. Oper. Res. 222 (1), 65–75.
Lu, C.C., Ying, K.C., Chen, H.J., 2016. Real-time relief distribution in the aftermath of disasters – A rolling horizon approach. Transp. Res. Part E 93, 1–20.
Meng, L.P., Kang, Q., Han, C.F., Zhou, M.C., 2017. Determining the optimal location of terror response facilities under the risk of disruption. IEEE Trans. Intell. Transp. Syst. 19 (2), 476–486.
Mete, H.O., Zabinsky, Z.B., 2010. Stochastic optimization of medical supply location and distribution in disaster management. Int. J. Prod. Econ. 126 (1), 76–84.
Moreno, A., Alem, D., Ferreira, D., 2016. Heuristic approaches for the multiperiod location-transportation problem with reuse of vehicles in emergency logistics. Comput. Oper. Res. 69, 79–96.
Rennemo, S.J., Fougner, K., Hvattum, L.M., Tirado, G., 2014. A three-stage stochastic facility routing model for disaster response planning. Transp. Res. Part E 62, 116–135.
Rockafellar, R.T., 1976. Monotone operators and the proximal point algorithm. SIAM J. Control Optim. 14 (5), 877–898.
Rockafellar, R.T., Wets, R.J.B., 1991. Scenarios and policy aggregation in optimization under uncertainty. Math. Oper. Res. 16 (1), 119–147.
Rottkemper, B., Fischer, K., Blecken, A., 2012. A transshipment model for distribution and inventory relocation under uncertainty in humanitarian operations. Socioecon. Plann. Sci. 46 (1), 98–109.
Sawik, T., 2014. Joint supplier selection and scheduling of customer orders under disruption risks: single vs. dual sourcing. Omega 43, 83–95.
Sheu, J.B., Pan, C., 2014. A method for designing centralized emergency supply network to respond to large-scale natural disasters. Transp. Res. Part B 67, 284–305.
Tofighi, S., Torabi, S.A., Mansouri, S.A., 2016. Humanitarian logistics network design under mixed uncertainty. Eur. J. Oper. Res. 250 (1), 239–250.
Veliz, F.B., Watson, J.P., Weintraub, A., R. Wets, J.-B., Woodruff, D.L., 2015. Stochastic optimization models in forest planning: a progressive hedging solution approach. Ann. Oper. Res. 232 (1), 259–274.
Wassenhove, L.N.V., 2006. Humanitarian aid logistics: supply chain management in high gear. J. Oper. Res. Soc. 57 (5), 475–489.
Wassenhove, L.N.V., Martinez, A.J.P., 2012. Using OR to adapt supply chain management best practices to humanitarian logistics. Int. Trans. Oper. Res. 19 (1-2), 307–322.
Watson, J.P., Woodruff, D.L., 2011. Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems. Comput. Manage. Sci. 8 (4), 355–370.
Zanjani, M.K., Nourelfath, M., 2014. Integrated spare parts logistics and operations planning for maintenance service providers. Int. J. Prod. Econ. 158, 44–53.
Zhang, J.H., Li, J., Liu, Z.P., 2012. Multiple-resource and multiple-depot emergency response problem considering secondary disasters. Expert Syst. Appl. 39 (12), 11066–11071.
Zhou, Y., Liu, J., Zhang, Y., Gan, X., 2017. A multi-objective evolutionary algorithm for multi-period dynamic emergency resource scheduling problems. Transp. Res. Part E 99, 77–95.
Zokaee, S., Bozorgi-Amiri, A., Sadjadi, S.J., 2016. A robust optimization model for humanitarian relief chain design under uncertainty. Appl. Math. Modell. 40 (17), 7996–8016.