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The active control of the lateral movement of a motor suspended under a high-speed locomotive

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Abstract

In order to improve the dynamic performance of high-speed locomotives, an active control method for the movement of a suspended motor is proposed, in which a damper is replaced by an actuator. A simplified mechanical model of the flexible suspension system is established to study its dynamics and create an active control strategy. Then, a multi-body dynamic model of the locomotive is simulated to obtain the conditions for optimal control. The results show that a smaller damping of the lateral suspension is conducive to improving the dynamic performance of the locomotive; the system has a theoretical optimum damping ratio of between 0.1 and 0.3. A small level of damping results in a large lateral displacement of the suspended system, which cannot be allowed to occur due to space limitations. The optimal control is used with the aim of reducing the lateral vibration amplitude of the suspended motor system and bogie frame. The active control of the suspension system improves the lateral dynamic performance of the locomotive and effectively limits the lateral displacement of the suspended system relative to the bogie frame. Compared with a passive suspension approach, the lateral forces on the wheelset are reduced by up to 25% at speeds between 140 and 180 km/h on straight track, and the nonlinear critical speed is increased from 390 to 480 km/h.

Keywords

High-speed locomotive, drive system, flexible suspension, active control, dynamic vibration absorber

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Introduction

The motor of a train can be flexibly suspended under the bogie frame or under the car body using swing rods or other flexible components. This is a new technology that has been introduced to high-speed train operation in recent years.[1–3] This approach can improve the lateral dynamic performance of locomotives or motorized cars at high running speeds, and it has been implemented on 160 km/h locomotives and high-speed motorized cars such as the CRH3, ''Blue Sword'' and the ''China Star'' in China. Many locomotives in Europe that are capable of a speed of 200 km/h are based on the E120 locomotive and their motors are beginning to be flexibly suspended.[2] Alfi et al.[4] have analyzed the effects of the way that the motor is connected on the critical speed of high-speed railway vehicles and concluded that an elastic connection is conducive to high stability of the system. Lou and Jin[5], Luo et al.[6] Zhang et al.[7] and Zhang et al.[8] have made significant contributions to the study of the dynamic behavior of high-speed locomotives with a flexibly suspended motor.

There is a large literature on the investigation of the mechanism of flexible suspension of drive systems and their effects on the lateral dynamics of a locomotive. Alfi et al.[4] showed that the mass of the motor acts as a dynamic absorber, and thus it affects bogie hunting in a way that improves vehicle stability. Yao et al.[9] showed that the lateral and yaw motions of the motor relative to the bogie frame decreases the lateral vibration energy of the bogie frame, this improves the lateral stability and reduces the lateral forces acting on the wheelset. These studies also showed that a smaller lateral damping of the system used to suspend the motor is conducive to improving the dynamic behavior of the vehicle. A laterally flexible suspension system decouples the inertia of the motor and bogie frame when a large damping is used.[5,6,9] The tight space under a train means that a high value of the lateral damping of the motor and a stopper are used to avoid the motor hitting the frame of the train.

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However, a large lateral damping of the motor suspension system results in a poor lateral dynamic performance of the locomotive.

Recently, a number of studies on active suspensions for the control of railway wheelsets have reported control technologies for primary suspensions that can eliminate the conflict between high-speed hunting stability and the curving performance of railway vehicles.[10–13] However, more detailed studies are required to overcome both technical and psychological obstacles before the railway industry can start adopting this emerging technology. The problems that need to be solved include operational concerns, such as safety and reliability, and the cost of the proposed systems.[11] The active control of a wheelset also has safety implications, thus it has a long way to go before it is widely adopted.

This paper considers an approach to the active control of the lateral movement of the system used to suspend the motor. The aim of this study is to reduce the amplitudes of lateral vibrations of the motor and the bogie frame. The lateral damper found in the suspension system of the motor is replaced by an actuator, however, all the other elements, such as the swing rods and rubber joints, are not changed. First, a simplified equivalent model is established and the control parameters of the optimal control are calculated. Then, a multi-body dynamics model of the whole locomotive is simulated with the lateral active control to verify the theoretical analysis. This study proposes a new method to limit the lateral movement of the suspension system of the motor for use on high-speed locomotives. The proposed approach has no safety implications and thus is attractive to industry.

The mechanism of the flexible suspension of the motor

Model of the dynamics of a flexible suspension system

For a high-speed locomotive, the motor is suspended under a bogie frame using a single rubber joint and two swing rods. The single rubber joint and the two swing rods are arranged on the two sides of the motor along the track direction. The two swing rods can be installed on the side of the motor or the non-motor end. Relative to the bogie frame, the drive system displays both lateral and yaw motions in the horizontal plane. In this study, a bogie is equipped with a Bo' axle arrangement and the effects due to car body movement are ignored. The bogie frame is assumed to be connected via the inertial coordinate of the secondary suspension. This simplified dynamics model is shown in Figure 1.

To calculate the mass and the yaw inertia of the flexible suspended part of the drive system, the non-flexible suspended part was added to the bogie frame.

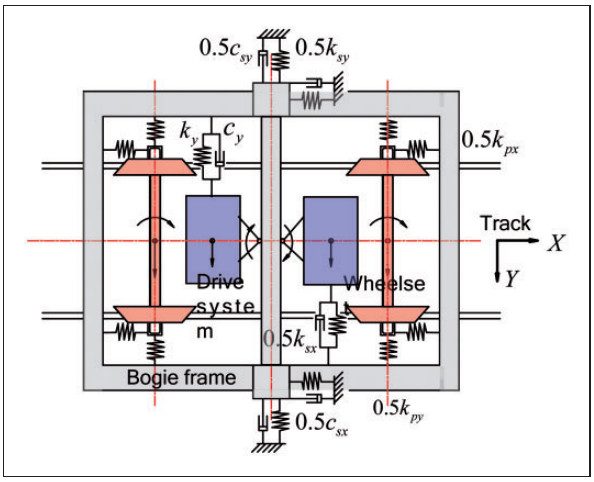

Figure 1. The model of a bogie supporting a flexible suspended drive system.

Taking the lateral and yaw movements of the bogie frame, the wheelset and the drive systems into account, a model of the dynamic system that included five rigid bodies and 10 degrees of freedom was established. The equation of motion is

$$M\ddot{X} + C\dot{X} + KX = Q \tag{1}$$

where M, C and K are the mass, damping and stiffness matrix of the system, respectively, and Q is the external force vector. The detailed matrices are shown in the Appendix and the parameters of this model are listed in Table 1.

For the motion equation, the lateral and yaw movements of the wheelset were coupled with the tangential forces of the wheel–rail contact. The lateral and yaw frequencies of the wheelset are equal, however, their phase angles are different. Ignoring spin creep forces, the stiffness matrix and damping matrix K_w and C_w of the linear dynamics equation of the wheelset positioned in a straight line are as follows

$$K_w = \begin{bmatrix} k_{py} + k_{gy} & -2f_\eta \\ 2\lambda_e l_0 f_\xi / r_0 & l_1^2 k_{px} + k_{g\psi} \end{bmatrix} \tag{2}$$

$$C_w = \frac{1}{v}\begin{bmatrix} 2f_\eta & \\ & 2l_0^2 f_\xi \end{bmatrix} \tag{3}$$

For the asymmetric stiffness matrix K_w, there are complex eigenvalues for the corresponding system matrix. If any real part of the eigenvalues is positive, the system is unstable. The non-diagonal entries in K_w represent the coupling of the wheelset's lateral and yaw movements, which imparts energy to the system. The C_w consumes energy and is inversely proportional to the speed v. In a period of hunting motion, if the input and the consumed energy are equal, the linear system is in the critical stability

Table 1. The parameters of the model.

Symbol	Definition	Value	Symbol	Definition	Value
m_w	Mass of wheelset	2585 kg	$2b$	Wheel base	2.5 m
I_w	Yaw inertia of wheelset	2024 kg.m^2	$2l_0$	Distance of contact spot	1493 mm
m_f	Mass of bogie (including non-flexible suspended part of drive system)	7186 kg	$2l_l$	Lateral spacing of primary suspension	2200 mm
I_f	Yaw inertia of bogie (including non-flexible suspended part)	9751 kg.m^2	$2l_2$	Lateral spacing of secondary suspension	2200 mm
m_d	Mass of flexible suspended part of drive system	1765 kg	r_0	Wheel rolling radius	0.625 m
I_d	Yaw inertia of flexible suspended part	1019 kg.m^2	λ_e	Wheel–rail contact conicity	0.15
k_{px}	Primary longitudinal stiffness per axle	40 kN/mm	f_ξ	Longitudinal creep coefficient	8.624e6
k_{py}	Primary lateral stiffness per axle	6 kN/mm	f_η	Lateral creep coefficient	8.144e6
k_{sx}	Secondary longitudinal stiffness	0.8 kN/mm	k_{gy}	Gravitational stiffness	41.4 kN/m
k_{sy}	Secondary lateral stiffness	0.8 kN/mm	$k_{g\psi}$	Gravitational angular stiffness	−23 kN/rad
c_{sx}	Secondary longitudinal damping	1000 N.s/m	k_r	Contact stiffness of wheel–rail	50 kN/mm
c_{sy}	Secondary lateral damping	60 kN.s/m	v	Max. running speed	200 km/h

state. The corresponding speed v is the linear critical speed.

The lateral and yaw vibrations of the drive system have the same frequency and attenuation coefficient. The equation for the lateral stiffness k_y and damping c_y of the suspended drive system are as follows

$$\begin{aligned} k_y &= (2\pi f_y)^2 m_d \\ c_y &= \xi_y 2m_d(2\pi f_y) \end{aligned} \tag{4}$$

where f_y is the lateral frequency of the suspended drive system (unit: Hz), ξ_y is the damping ratio of the suspended drive system, m_d is the mass of the suspended parts of the drive system (unit: kg). Similarly, we can obtain expression for the yaw stiffness k_ψ and damping c_ψ between the driving system and bogie frame.

Effect of the flexible suspension on the linear stability of the bogie

For a rigid wheelset of a rail vehicle, the wheelset exhibits a hunting motion on the track due to the conicity of the wheel tread. Due to the stiffness of the primary guide and the effect of gravity, the hunting motion of the wheelset is stable over a certain range of speeds. That is, the wheelset tends to be attracted towards the center of the track. When the vehicle speed exceeds a certain value (called the critical speed), the maximum lateral displacement of the wheelset has a non-zero value (the wheel–rail gap usually). When the lateral displacement of the wheelset cannot quickly converge, the effect is called vehicle lateral instability. Changes in the values of some parameters, such as an increase in the wheel conicity due to wear, result in a considerably reduced level of lateral stability of the vehicle. This lateral instability degrades the dynamic performance of the vehicle and in some cases it can cause a derailment. Thus, a rail vehicle requires its lateral stability to be routinely checked, to ensure a large margin of safety. The lateral stability of a rail vehicle is one of the most important and difficult problems in the field of vehicle system dynamics.[14]

Checking the lateral stability of a vehicle can involve both linear and nonlinear methods. A linear stability analysis can be used to estimate a vehicle's critical speed by quickly optimizing parameters; the instability mode can be identified from the corresponding mode shapes.

The eigenvalues and eigenvectors of the homogeneous items of equation (1) can be calculated in the state space for a non-symmetric stiffness and non-proportional damping matrix. The imaginary and real part of each eigenvalue denotes the circular frequency of the vibration and its attenuation coefficient. The eigenvectors denote the rigid mode shapes of the corresponding frequency. The root locus curves of a linear system consist of all the eigenvalues at a speed. If all the real parts of the eigenvalues are negative, the motion of this system is stable.

In order to discuss the effect of the structure and the suspension parameters of the drive systems on the lateral stability of the bogie, the linear stability of four types of models are analyzed.

1. No driving system model.
2. The driving system is fixed to the bogie frame in a non-flexible manner.
3. A single wheelset with elastic positioning, which is equivalent to the bogie frame having only one degree of freedom in the longitudinal direction.
4. The drive system is flexibly suspended under the bogie frame.

Figure 2 shows the calculated linear critical speeds for different models and suspension parameters of the drive system. The lateral stability of the drive system has its worst value when it is non-flexibly fixed to the

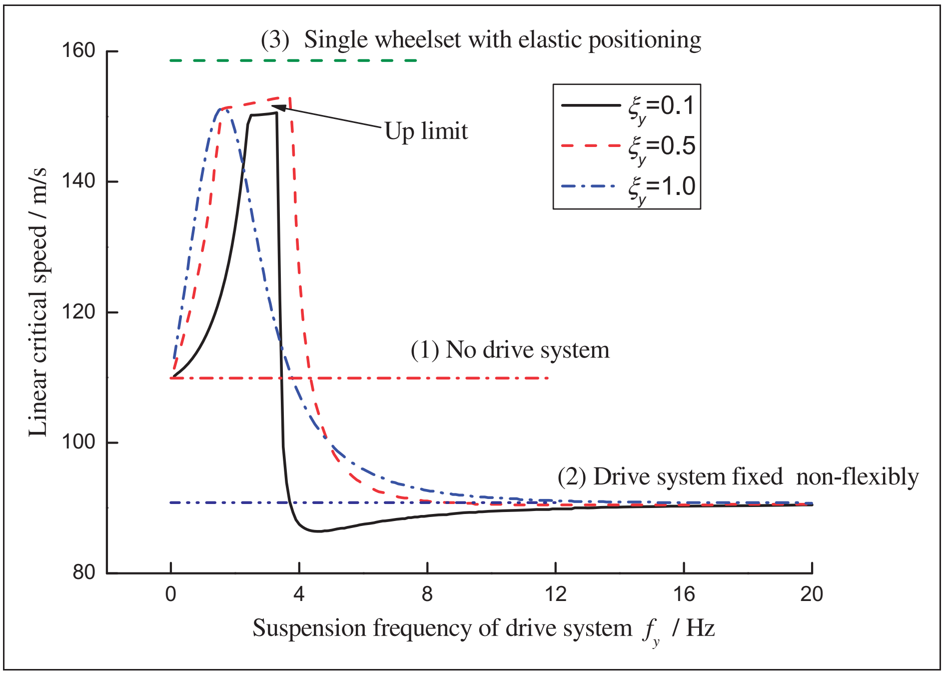

Figure 2. The linear critical speed.

bogie frame, this is due to the drive system increasing the mass and yaw inertia of the bogie frame. A single wheelset with flexible positioning results in the bogie frame not participating in a hunting motion, and its linear critical speed thus has its largest value. The critical speeds of the bogie with a flexibly suspended system are related to the frequency of the suspension f_y. When f_y is zero, it is equivalent to no drive system. A large f_y means that the drive system is fixed to the bogie frame in a non-flexible manner. When f_y is close to the hunting frequency of the wheelset, the lateral stability of the bogie with a flexibly suspended drive system is significantly better than for no drive system, and the critical speed is related to the suspension damping ξ_y. The flexible suspension of the drive system is equivalent to a dynamic vibration absorber and its lateral and yaw vibrations absorb the vibration of the bogie frame. When f_y is close to the excitation frequency, the vibration amplitude of the bogie frame is at its minimum value and that enhances the lateral stability of the bogie.

Effect of a flexible suspension

There is a large dynamic force between the wheel and rail due to track irregularities. The lateral forces on the wheelset (the sum of the guiding forces on the two wheels on an axle) are an important indicator of a vehicle's dynamic performance. As the lateral forces on the wheelset increase with an increase in the running speed, low dynamic forces are emphasized particularly in high-speed vehicles. In order to study the effects of the flexible suspension system on the lateral forces experienced by the wheelset, the dynamic model of the bogie in equation (1), which takes a horizontal track irregularity and the wheel–rail gap into account was simulated. The flange contact was taken to be the unilateral contact stiffness k_r and it was damped in the simulation. The Runge–Kutta method was used to obtain numerical results and a statistically valid value of 2.2-times the standard deviation was used to evaluate the maximum lateral forces experienced by the wheelset.

The simulation results obtained for various values of the suspension frequencies f_y and for a straight track containing an AAR5 class irregularity are shown in Figures 3 and 4. The effect of the flexible suspension on the lateral forces acting on the wheelset is the same as for the lateral stability. The flexible suspension of the drive system reduces the lateral forces acting on the wheelset and the lateral displacement of the bogie frame. The lateral forces on the wheelset and the bogie frame displacement have their lowest value when the suspension frequency is close to the hunting frequency. As shown in Figure 4, a smaller suspension damping ξ_y is beneficial in reducing the lateral forces acting on the wheelset. If ξ_y has a large value, the flexible suspension can be seen to decouple the inertia of the drive system and bogie frame, which improves the performance of the vehicle.

Based on the presented analysis, it can be inferred that an increases in the equivalent conicity of the wheel, just as for wear or another tread, increases the hunting frequency of the rail vehicle and the optimal frequency or stiffness of the suspended drive system has to be increased to improve the lateral dynamic performance of the vehicle, especially at high speeds.

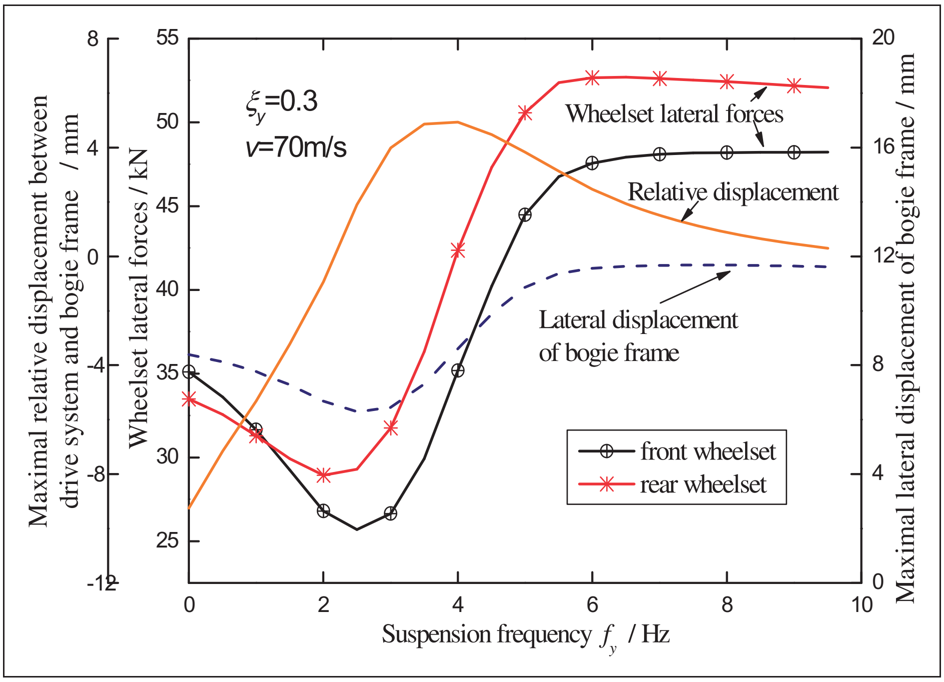

Figure 3. The effect of ξ_y on the lateral forces experienced by the wheelset and the lateral displacement of the bogie frame.

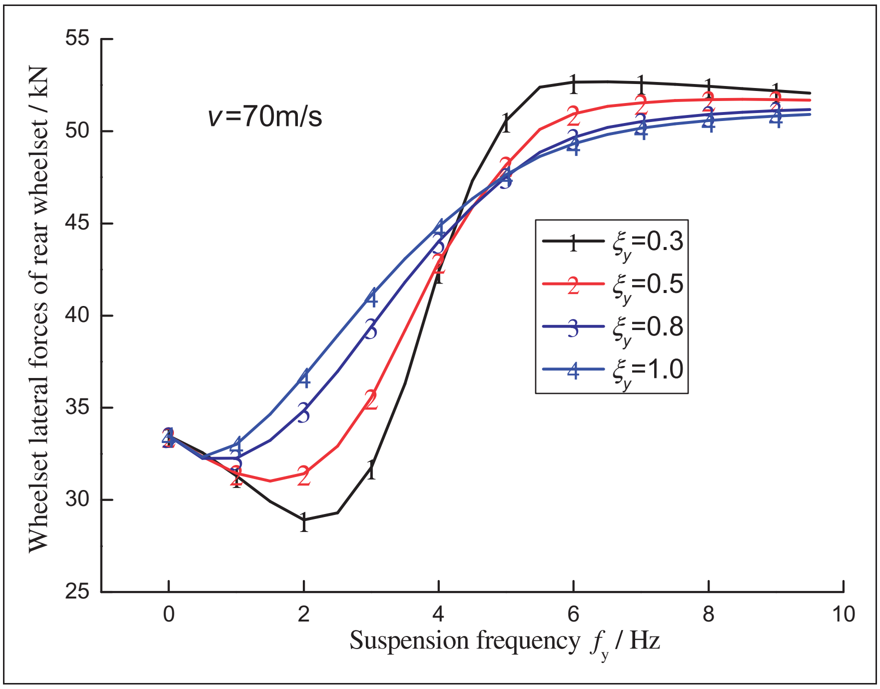

Figure 4. The effect of ξ_y on the lateral forces experienced by the wheelset.

The active control of the lateral movement of the flexibly suspended system

Studies have shown that a smaller damping in the lateral direction of the suspended drive system can help improve the lateral dynamic performance of a locomotive. However, a smaller damping in the lateral direction results in a large displacement in that direction by the suspended system. However, the lateral displacement of the suspended system needs to be limited due to the limited amount of space under a train. In practice, impacts between the suspended system and the structure of the train are avoided by increasing the lateral damping and also using a stopper. However, a large damping ratio can lead to a deteriorated dynamic performance of the train. Instead of the lateral damper that is normally used in the suspension system, this paper proposes the use of an actuator that is used in an active control approach. Except for replacing the lateral damper by an actuator, the rest of the system remains the same.

In order to achieve active control of the lateral movement of the suspended drive system, a simplified equivalent model of the lateral movement of the bogie and drive system was created, as shown in Figure 5. The degrees of freedom and the parameters used in this model were based on equivalent lateral movements of the bogie. Assuming the lateral position of the car body is fixed, the lateral degrees of freedom of the bogie frame and driving system can be taken into account. A simple harmonic motion of the wheelsets was applied as the source of system excitation.

The equation of motion of the equivalent model can be written as

$$m_f\ddot{y}_f + \left(k_{sye} + k_{py} + k_y\right) y_f - k_y y_d = -u + k_{py} y_w \tag{5}$$

$$m_{de}\ddot{y}_d + \left(k_y + k_{ce}\right) y_d - k_y y_f = u + k_{ce} y_w \tag{6}$$

where k_{py} is the primary lateral stiffness, k_{ce} is the equivalent combined lateral stiffness of the transmission of the double-hollow shaft, k_{sye} is the equivalent secondary lateral stiffness, m_{fe} is the equivalent mass of the bogie frame, m_{de} is the equivalent mass of the flexibly suspended parts of the drive system and, u is the output force of the actuator.

The state variables were defined as follows

$$x = \left\{ y_f \quad y_d \quad \dot{y}_f \quad \dot{y}_d \right\}^T \tag{7}$$

The state equation of system can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu + F \\ y &= Cx \end{aligned} \tag{8}$$

where A, B, F, C are either a system matrix or vector.

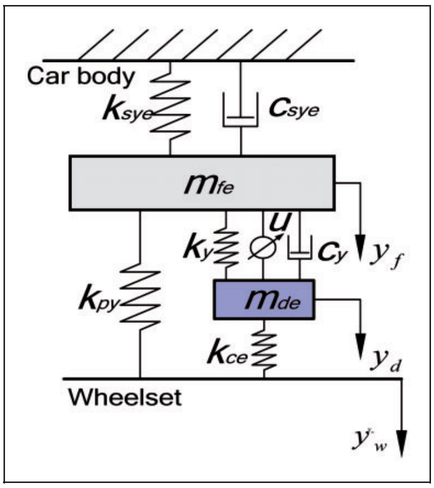

Figure 5. The simplified model of the railway vehicle.

Based on optimal control theory, the control signal $u(t)$ should follow following function in order to reach its minimum value

$$J = \frac{1}{2}\int_0^\infty \left(x^T Q x + R u^2\right) \mathrm{d}t \tag{9}$$

where Q and R are weight coefficient matrices. An increase in R means a decrease in the input energy of the actuator. The larger a coefficient in the matrix Q, the smaller is the vibration energy of the corresponding physical parameter. The coefficients in the Q and R matrices were adjusted in the simulation to obtain a better control performance. The principle behind the selection of the weight coefficient is to reduce the lateral displacement of the drive system and bogie frame. The optimal feedback gain matrix K was calculated by solving the Riccati equation, which was solved by the command $[K,P,E]=$ lqr(A,B,Q,R) in Matlab. The output force of the actuator was obtained from the following equation

$$u = -K \cdot \mathrm{x} \cdot x \tag{10}$$

The multi-body dynamic model simulations and tests

In order to verify the influence of a flexible suspension of the drive system on the dynamic performances of a locomotive, and to simulate the improvement of the dynamic performance obtained using active control, a dynamic model of a 200 km/h class locomotive that operates in China was created in SIMPACK, and it is shown in Figure 6. The model consists of 47 rigid bodies and includes 172 degrees of freedom. The locomotive utilizes the suspended drive system mentioned above and it has a JM3 wear tread on its wheels. The locomotive's main parameters are listed in Table 2. The track irregularities were taken to be German high class defects and the space wavelength ranged between 0 and 139 m. The hunting frequency of the wheelset was 2 Hz at a speed of 200 km/h.

Effects of the parameters of the suspended drive system

The influence of the length of the swing rod and the lateral damping of the suspended system on the dynamic performance of the locomotive was simulated. Figure 7 shows the effect of the length of the swing rod on the lateral forces experienced by the wheelset. The lateral forces are reduced with an increase in the length of the swing rod. The curves in Figure 8 show the results of the simulations, including the maximum lateral forces on the wheelsets and the lateral displacements. The largest lateral force is located at the rear wheelset. The simulation results show that the lateral force on the wheelset has its

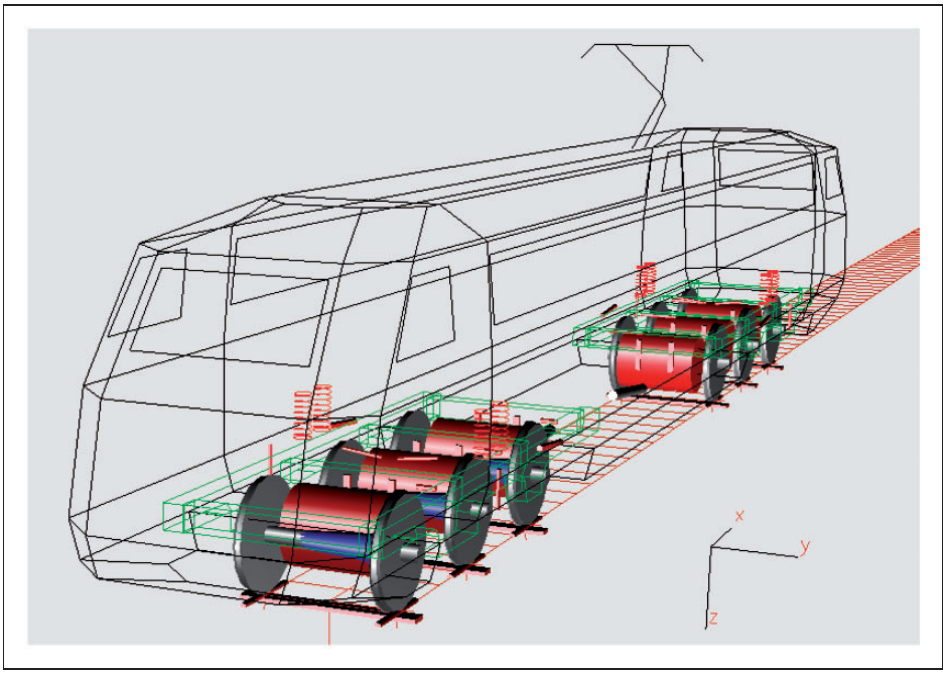

Figure 6. The multi-body dynamic model of the locomotive.

Table 2. The main parameters of the locomotive.

Axle arrangement	Axle load (t)	Motor arrangement	Wheel base (m)	Bogie distance (m)	Axle guidance stiffness (signal axlebox) (kN/mm)		Secondary lateral stiffness (one side of a bogie) (kN/mm)
					Longitudinal	Lateral	
$2C_O$	21	Sequence	2.0–2.0	13.05	28.67	3.67	0.428

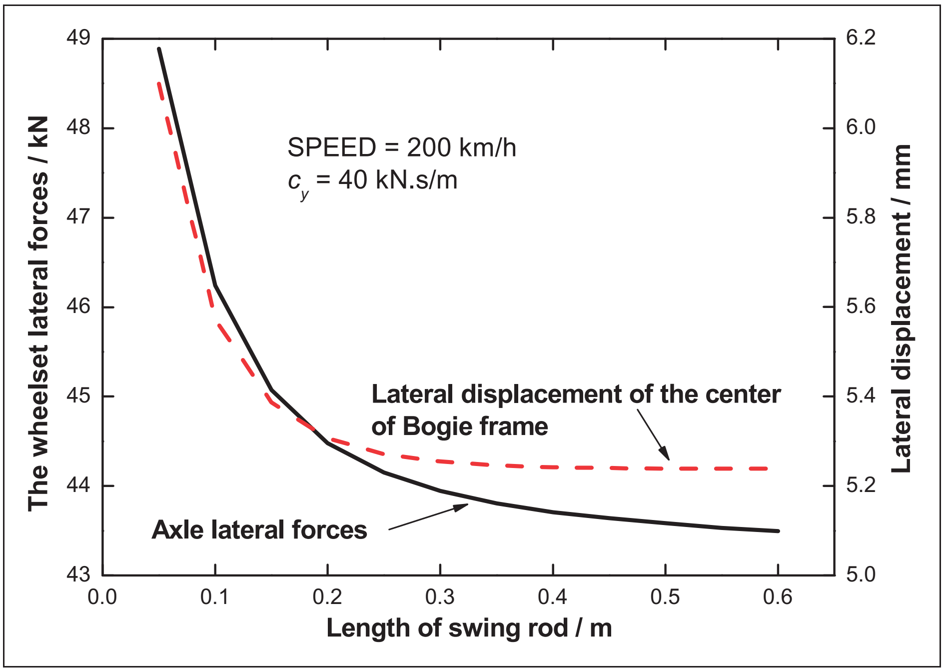

Figure 7. The effect of the length of the swing rod on the lateral forces experienced by the wheelset.

minimum value when the lateral damping ratio of the driving system has a value between 0.1 and 0.3 and the lateral force on the wheelset increases with an increase in the lateral damping. The largest lateral displacement between the drive system and the bogie frame occurs for a small lateral damping of the drive system, which cannot be achieved in reality. The relative displacement from the drive system towards the

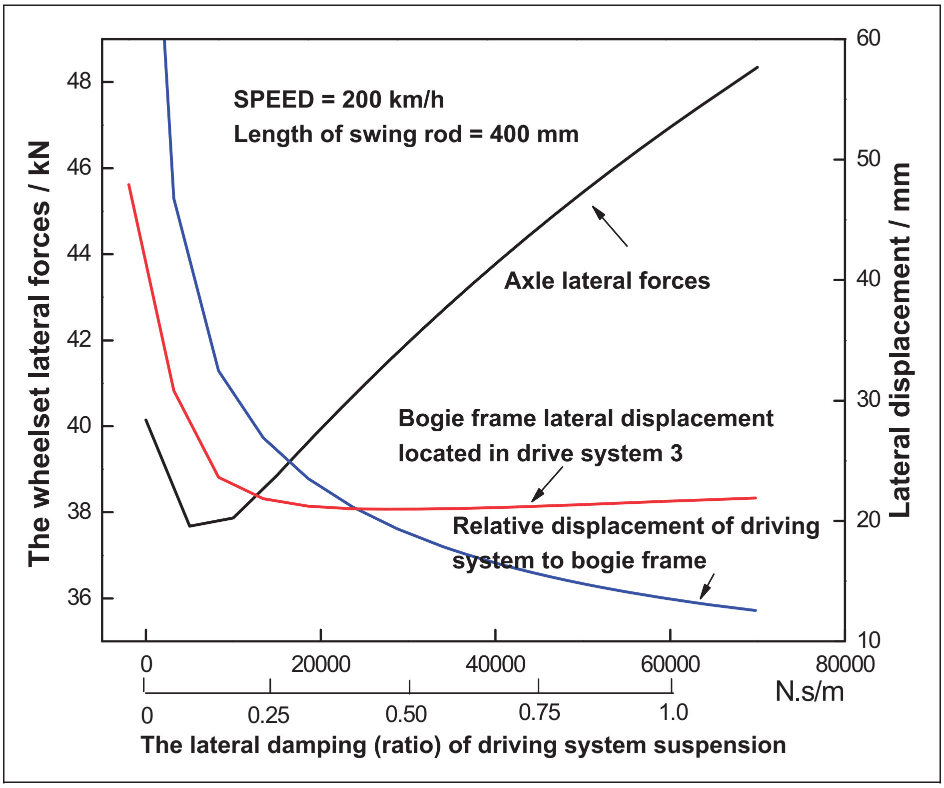

Figure 8. The effect of the lateral damping on the lateral dynamic performance.

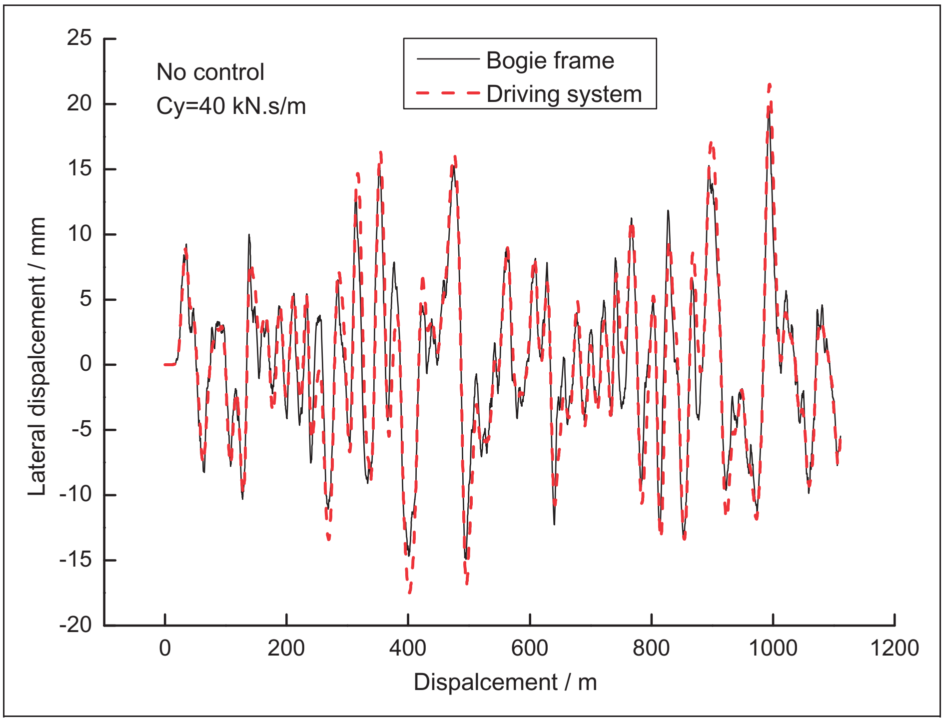

Figure 9. The lateral displacement of the drive system with a passive suspension.

bogie frame is minimized when the lateral damping increases. The behavior of the lateral forces on the wheelset and the lateral displacement of the bogie frame with a change in the lateral damping are basically the same. The function of the drive system as a dynamic vibration absorber is not obvious at a larger value of the damping. To improve the lateral dynamic performance of a high-speed locomotive, effective measures must be taken to reduce the lateral vibration of the bogie frame.

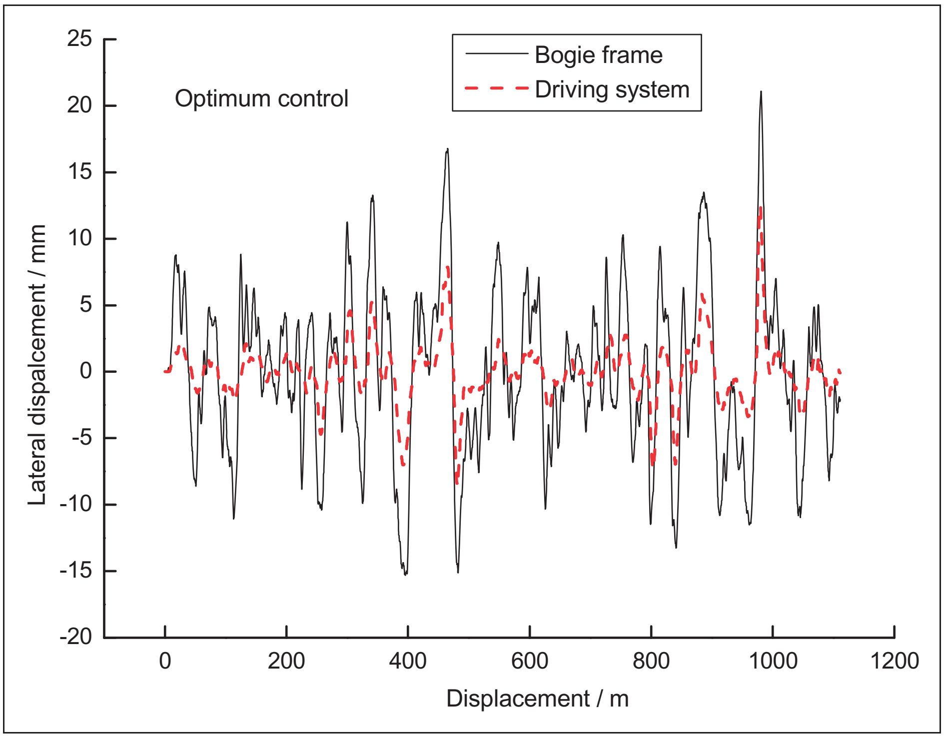

Figure 10. The lateral displacement of the drive system under optimal control.

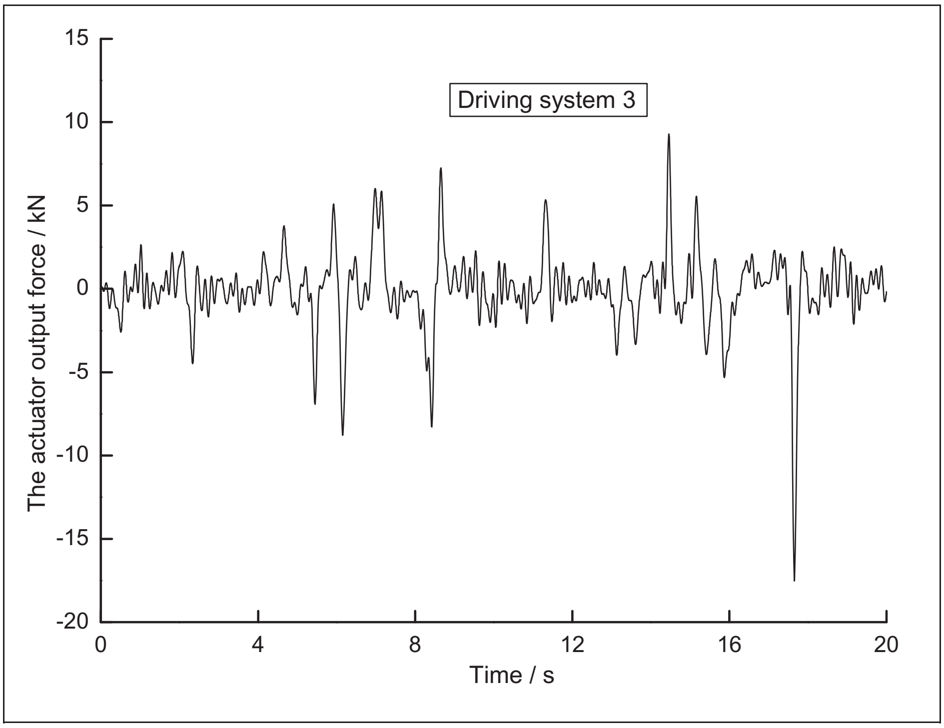

Figure 11. The output force of the actuator.

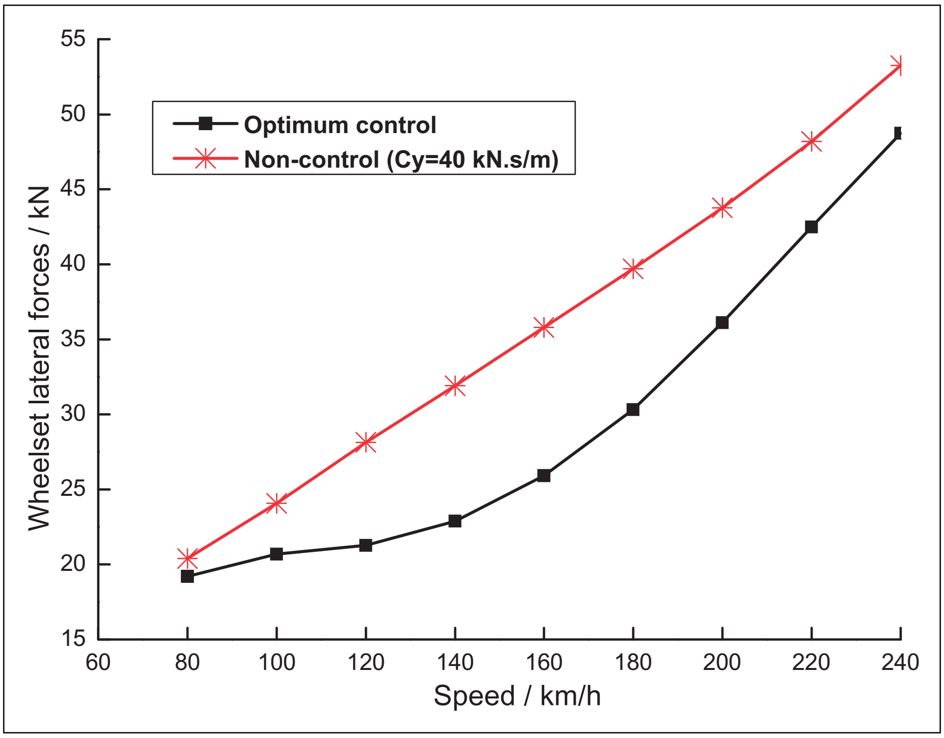

Figure 12. Comparison of the lateral forces experienced by the wheelset.

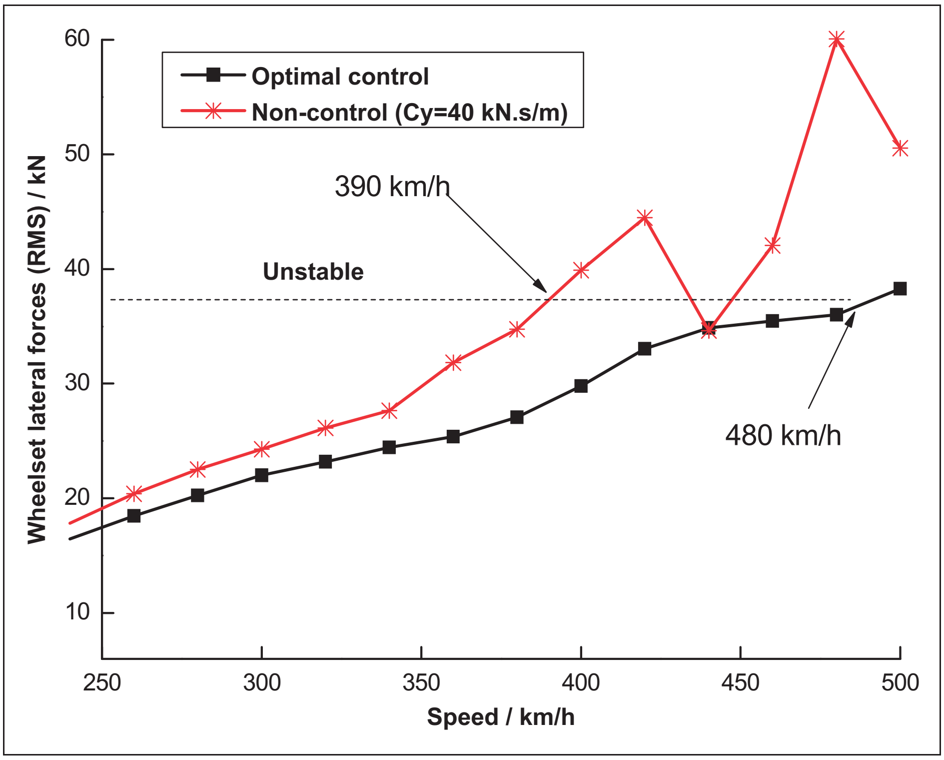

Figure 13. The lateral stability of the locomotive.

Results of the simulations that considered active control

Based on the equivalent model of the suspended drive system and the previously discussed optimal control strategy, the control of the output forces of the actuator can be calculated in real-time. As previously noted, the R and Q are the weight coefficient matrices. An increase in a weight coefficient in R means a decrease in the input energy of the actuator. The larger the weight coefficient in matrix Q, the smaller is the vibration energy of the corresponding physical parameter. The coefficients in the Q and R matrices were adjusted in the simulation to obtain a better control performance. A value of $R = 1e\text{-}5$ and diagonal coefficients of the matrix Q of [0.5, 1, 0, 0] were used in the simulations. A response delay of 0.02 s of the control system was considered in the simulation. The lateral displacements of the bogie frame and the drive system using the optimal control and no-control on straight track at a speed of 200 km/h are shown in Figure 9 and 10, respectively. These curves show that the active control approach can limit the lateral movement of the drive system.

The 2.2-times standard deviation values used to indicate the maximum statistically valid values of the lateral displacement of the drive system are 14.96 and 5.06 mm, respectively. If the weighting coefficients in Q are large, the relative displacement from the drive system to the bogie frame is small, and this outcome is unwanted as it deteriorates the lateral dynamic performance of the vehicle. Figure 11 shows the output force of the actuator of drive system 3, the maximum output power is less than 20 kN. Figure 12 shows the lateral forces acting on the wheelset with the passive suspension and active control as a function of speed. It can be seen that the active control reduces the lateral forces acting on the wheelset. The lateral forces are decreased by 25% at speeds between 140 and 180 km/h. Figure 13 shows results obtained for the lateral stability of the locomotive. The nonlinear critical velocity of the locomotive was calculated using the procedure in the standard EN14363.[14,15] If the root-mean-squared values of the lateral forces acting on the wheelset exceed the limiting value, the locomotive is considered to be laterally unstable and the corresponding speed is the nonlinear critical velocity. Compared with the passive suspension, the nonlinear critical speed of the locomotive used active control increases from 390 to 480 km/h.

Conclusions

1. A flexible suspension of the drive system can effectively improve the lateral dynamic performance of a high-speed locomotive. The optimal lateral damping ratio of the drive system is between 0.1 and 0.3. It causes a larger lateral relative displacement of the drive system towards the bogie frame, which can not be achieved in an actual structure.
2. The active control of the lateral suspension of the drive system is proposed in order to reduce the amplitude of the lateral vibration and the vibration of the bogie frame. The damper normally found in the suspension system is replaced by an actuator, with the other components, such as the hanging rods and the rubber joint, being retained.
3. The simulation results show that the active control of the lateral movement of the suspended drive system can significantly improve the lateral dynamics performance of the locomotive and limit the lateral movement of the drive system. Compared with a passive suspension, the maximum lateral forces experienced by a wheelset under active control are reduced by up to 25% at speeds between 140 and 180 km/h, and the nonlinear critical speed of the locomotive increases from 390 to 480 km/h.

Declaration of Conflicting Interests

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References

1. Middengorf E. The new electric locomotive generation for DB AG. *Electric Traction J* 1997; 19: 1–11.
2. Christian S. The 12X high power locomotive. *Rail Int* 1996; 7: 24–30.
3. Hödi H and Haigermoser A. Development of new motor bogie concept for high performance locomotives. *Electric Tract J* 1995; 17: 69–77.
4. Alfi S, Mazzola L and Bruni S. Effect of motor connection on the critical speed of high-speed railway vehicles. *Veh Syst Dyn* 2008; 46: 201–214.
5. Luo Y and Jin C. Research on the rules of suspension parameters to driving equipments suspended in bogie frames. *China Railway Sci* 2007; 28: 78–82.
6. Luo Y, Luo S and Jin D. Research on driving equipment suspension parameters and structures of locomotives with motor elastically mounted on the frame. *China Railway Sci* 2005; 26: 57–61.
7. Zhang HJ, Cheng XH, Sun YP, et al. The analysis of bogie platform of 200 km/h high-speed passenger locomotive in our country. *J China Railway Soc* 2007; 29: 101–106.
8. Zhang HJ, Huang YM, Li GJ, et al. Research on the speed-increase six-axle locomotive with driving equipment flexibly suspended on the frame. *China Railway Sci* 2008; 29: 84–88.
9. Yao Y, Zhang K, Luo SH, et al. The dynamic effect mechanism of driving system elastically mounted on the bogie frame to locomotive. *J Vib Engng* 2012; 25: 481–487.
10. Goodall RM and Kortum W. Mechatronic developments for railway vehicles of the future. *Control Engng Pract* 2002; 10: 887–898.
11. Mei TX and Goodall RM. Recent development in active steering of railway vehicles. *Veh Syst Dyn* 2003; 39: 415–436.

12. Pearson JT, Goodall RM, Mei TX, et al. Active stability control strategies for a high speed bogie. *Control Engng Pract* 2004; 12: 1381–1391.
13. Bruni S, Goodall RM, Mei TX, et al. Control and monitoring for railway vehicle dynamics. *Veh Syst Dyn* 2007; 45: 743–779.
14. Polach O. On non-linear methods of bogie stability assessment using computer simulations. *Proc IMechE, Part F: J Rail Rapid Transit* 2006; 220: 13–27.
15. Yao Y, Zhang HJ, Li YM, et al. The dynamic study of locomotives under saturated adhesion. *Veh Syst Dyn* 2011; 49: 201–214.

Appendix

$$M = \begin{bmatrix} m_w &&&&&&&&& \\ & I_w &&&&&&&& \\ && m_w &&&&&&& \\ &&& I_w &&&&&& \\ &&&& m_f &&&&& \\ &&&&& I_f &&&& \\ &&&&&& m_d &&& \\ &&&&&&& I_d && \\ &&&&&&&& m_d & \\ &&&&&&&&& I_d \end{bmatrix}$$

$$K = \begin{bmatrix} k_{py}+k_{gy} & -2f_\eta & & & -k_{py} & -bk_{py} & & & & \\ \frac{2\lambda_e l_0 f_\xi}{r_0} & 2l_1^2k_{px}+k_{g\psi} & & & & -2l_1^2k_{px} & & & & \\ & & k_{py}+k_{gy} & -2f_\eta & -k_{py} & bk_{py} & & & & \\ & & \frac{2\lambda_e l_0 f_\xi}{r_0} & 2l_1^2k_{px}+k_{g\psi} & & -2l_1^2k_{px} & & & & \\ -k_{py} & & -k_{py} & & \left(2k_{py}+k_{sy}+2k_y\right) & & -k_y & & -k_y & \\ -bk_{py} & -2l_1^2k_{px} & bk_{py} & -2l_1^2k_{px} & & \left(l_2^2k_{sx}+4l_1^2k_{px}+4b^2k_{py}+2k_\psi\right) & & -k_\psi & & -k_\psi \\ & & & & -k_y & -lk_y & k_y & & & \\ & & & & & -k_\psi & & k_\psi & & \\ & & & & -k_y & lk_y & & & k_y & \\ & & & & & -k_\psi & & & & k_\psi \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{2f_\eta}{v} & & & & & & & & & \\ & \frac{2l_0^2f_\xi}{v} & & & & & & & & \\ & & \frac{2f_\eta}{v} & & & & & & & \\ & & & \frac{2l_0^2f_\xi}{v} & & & & & & \\ & & & & c_{sy}+2c_y & & -c_y & & -c_y & \\ & & & & & l_2^2c_{sx}+2c_\psi & & -c_\psi & & -c_\psi \\ & & & & -c_y & -lc_y & c_y & & & \\ & & & & & -c_\psi & & c_\psi & & \\ & & & & -c_y & lc_y & & & c_y & \\ & & & & & -c_\psi & & & & c_\psi \end{bmatrix}$$

$$A = \begin{bmatrix} & & 1 & \\ & & & 1 \\ \frac{-\left(k_{sye}+k_{py}+k_y\right)}{m_{fe}} & \frac{k_y}{m_{fe}} & \frac{-\left(c_{sye}+c_y\right)}{m_{fe}} & \frac{c_y}{m_{fe}} \\ \frac{k_y}{m_{de}} & \frac{-\left(k_y+k_{ce}\right)}{m_{de}} & \frac{c_y}{m_{de}} & \frac{-c_y}{m_{de}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & -\frac{1}{m_{fe}} & \frac{1}{m_{de}} \end{bmatrix}^T$$

$$F = \begin{bmatrix} 0 & 0 & \frac{k_{py}y_w}{m_{fe}} & \frac{k_{ce}y_w}{m_{de}} \end{bmatrix}^T$$