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The Motor Active Flexible Suspension and its Dynamic Effect on the High-speed Train Bogie

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We put forward the motor active flexible suspension and investigate its dynamic effects on the high-speed train bogie. The linear and nonlinear hunting stability are analyzed using a simplified 8 degrees of freedom bogie dynamics with partial state feedback control. The active control can improve the function of dynamic vibration absorber of the motor flexible suspension in a wide frequency range, thus increasing the hunting stability of the bogie at high speed. Three different feedback state configurations are compared and the corresponding optimal motor suspension parameters are analyzed with the multi-objective optimal method. In addition, the existence of the time-delay in the control system and its impact on the bogie hunting stability are also investigated. The results show that the three control cases can effectively improve the system stability and the optimal motor suspension parameters in different cases are different. The direct state feedback control can reduce corresponding feed state's vibration amplitude. Suppressing the frame's vibration can significantly improve the running stability of bogie. However, suppressing the motor's displacement and velocity feedback are equivalent to increasing the motor lateral natural vibration frequency and damping, separately. The time-delay over 10 ms in control system reduces significantly the system stability. At last, the effect of preset value for getting control gains on the system linear and nonlinear critical speed is studied.

1 Introduction

Much research has been done about the active control of railway vehicles since 1980s. The tilting train, active radial steering and bogie stabilization have been widely studied [1–3]. With the continuous improvement of the vehicle running speed, active stability attracts more and more attention [4–6]. Since the effect of bogie primary suspension on the critical velocity of railway vehicles has been well studied. Many different approaches for active primary suspensions have been reported in the literature [7–9]. Active stability control used in bogie system can be applied either directly or indirectly [10]. The direct control involves the application of an actuator onto a solid wheelset or onto an axle with independently rotating wheels. A practical compromise is to apply a yaw torque indirectly at the secondary suspension level, a concept called 'Secondary Yaw Control' (SYC). The actuators may be designed to replace the traditional passive yaw dampers so that active control of the running gear can be introduced without a substantial redesign of the bogie, although this means that radical simplifications of the vehicle design may be limited [11].

Active stability is not only used for the newly-made railway vehicles for compromising the conflict between the hunting stability and the curving performance. With the increase in service life and mileage, the vehicle structure parameters and suspension performances may change due to the tread wear and component failure, causing the reduction of hunting stability. Loss of lateral stability results in poor ride comfort, huge wheel/rail action force or even derail accident. Active controls in high-speed trains are great alternatives of passive methods, which can compensate for the variation of the parameters of the vehicle and guarantee the running performances. There are a lot of literature on using active control onto the solid wheelset or independently rotating wheel to improve the stability of the bogie or wheelset, meanwhile, the control schemes also improve the curving performance. However, the application of active control onto wheelset directly to improve the running stability still has many challenges in practice.

The drive system or motor is suspended flexibly under the bogie frame or under the car body through the swing rods or other elastic. The active mass damping [12] or tuned vibration absorbers [13] have been applied in civil engineering structure widely, which inspires us to adopt the motor active flexible suspension to improve the vehicle lateral dynamics. The author found the active control of the lateral movement of the suspended drive system using SIMPACK can significantly improve the lateral dynamics performance of the locomotive, of which the maximum lateral forces experienced by a wheelset under active control are reduced by up to 25% [14]. In this paper, we continue some theoretical researches with a simplified bogie model. An indirec-

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t method for improving the bogie hunting stability through controlling the motor's and the frame's lateral movement was put forward. It does not change the main bogie structure, just need to install the actuator directly between the motor and the bogie frame. It was called as the active flexible suspension, which has the advantages of high feasibility and reliability, as well as low risk. Three different state feedback configurations are proposed and the corresponding optimal motor suspension parameters are compared. The bogie is not a completely controllable and observable system, a partial subsystem composed of a frame and two motors is chosen as the control model to achieve the vibration control. The vibration states of frame and motor need to be detected rather than the wheelset. To consider the inherited confliction between the system relative stability and control energy, a multi-objective optimization is used in the control design. Also, the linear and nonlinear stability of the bogie control system at different running speeds for each control configuration is investigated. Furthermore, the effects of time-delay in control system on the bogie system performances are discussed.

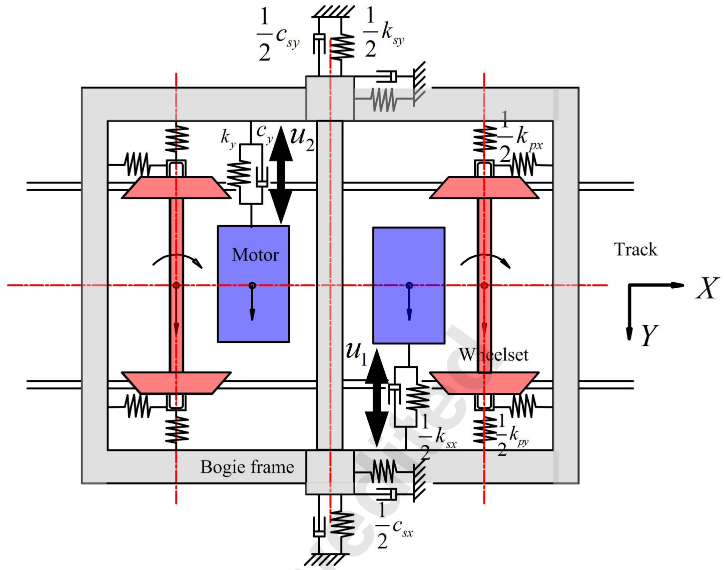

Fig. 1. The dynamics model of high-speed train bogie with motor active flexible suspension .

2 The motor active flexible suspension

2.1 The bogie linear dynamics model

The motor or driving system in some high-speed train bogies is connected flexibly to improve the railway vehicle hunting stability at high speed. This study proposes the way of using active control for the motor flexible suspension to further improve vehicle lateral dynamics performances [15]. In order to research the motor active flexible suspension, a simplified model of this system is shown in Figure 1. The model was found to have 5 rigid bodies and 8 degrees of freedom. The model describes the lateral movements of the bogie and reads,

$$\mathbf{M}\ddot{\mathbf{y}}+\mathbf{C}\dot{\mathbf{y}}+\mathbf{K}\mathbf{y}=\mathbf{q}+\mathbf{E}\mathbf{u}+\mathbf{w}d(t) \tag{1}$$

where $\mathbf{y}=[y_{w1},\varphi_{w1},y_{w2},\varphi_{w2},y_f,\varphi_f,y_{m1},y_{m2}]$. The states y_{w1}, y_{w2}, y_f, y_{m1}, and y_{m2} are the lateral displacements of wheelset 1, wheelset 2, frame, motor 1 and motor 2, respectively. φ_{w1}, φ_{w2}, and φ_f represent the yaw angles of wheelset 1, wheelset 2, and frame. **M**, **C**, and **K**, are the mass, damping and stiffness matrices of the bogie system. $\mathbf{u}=[u_1,u_2]^T$ is a vector of the lateral active control forces applied on the bogie frame by two inertial actuators located on the front and rear beams, respectively. **E** is the control influence matrix. In order to study the effects of active control on improving the bogie hunting stability, the damping of yaw damper is chosen to be one fourth of its design value such that the bogie is in a faulty state. The vector **w** is defined as $\mathbf{w}=[k_{py},0,k_{py},0,0,0,0,0]^T$. $d(t)$ is the external disturbance on the wheelsets due to, for example, track irregularity. In order to better understand the motor flexible suspension mechanism, the lateral stiffness and damping of the motor suspension are replaced by motor suspension frequency f_{my} and suspension damping ratio ξ_{my}. The parameters of the model were described in the reference [16].

The state equation of the system can be written as

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{F}d(t) \tag{2}$$

where the state vector is defined as

$$\begin{aligned}\mathbf{x}=[&y_{w_1},\varphi_{w_1},y_{w_2},\varphi_{w_2},y_f,\varphi_f,y_{m_1},y_{m_2},\\ &\dot{y}_{w_1},\dot{\varphi}_{w_1},\dot{y}_{w_2},\dot{\varphi}_{w_2},\dot{y}_f,\dot{\varphi}_f,\dot{y}_{m_1},\dot{y}_{m_2}]^T\end{aligned} \tag{3}$$

The **A**, **B**, and **F** are system matrice, control matrice and disturbance influence vector.

Consider a partial state linear feedback control $\mathbf{u}=-\mathbf{K}_c\cdot\mathbf{x}_c(t)$. Where, $\mathbf{K}_c$ is the feedback gain matrix and $\mathbf{x}_c$ is the partial feedback states. For the flexible suspension motor in lateral direction, a retractable actuator can be installed between the motor and the side beam of frame to achieve the frame vibration control as shown in Figure 1. In order to compare the control effects of different feedback states, we consider three different feedback states $\mathbf{x}_c$. They are states of the frame, the motor, and the frame together with the motor, separately. For the three cases, the $\mathbf{x}_c$ can be expressed as following,

$$\mathbf{x}_c=\begin{cases}[y_f,\varphi_f,\dot{y}_f,\dot{\varphi}_f], & \text{for case(1)}\\ [y_{m_1},y_{m_2},\dot{y}_{m_1},\dot{y}_{m_2}], & \text{for case(2)}\\ [y_f,\varphi_f,y_{m_1},y_{m_2},\dot{y}_f,\dot{\varphi}_f,\dot{y}_{m_1},\dot{y}_{m_2}], & \text{for case(3)}\end{cases} \tag{4}$$

Using this control law, the closed-loop dynamic matrix of the system reads

$$\mathbf{A}_{cl}=\mathbf{A}-\mathbf{B}\left[\,\mathbf{K}_c\ \ 0\,\right]. \tag{5}$$

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We can try to adjust some feedback gain coefficients to stabilize the closed-loop system $\mathbf{A}_{cl}$. Nevertheless, the stability margin and energy efficiency of the controller should be also taken into account in the design. The reasonable trade-off can be provided by the multi-objective optimal method. These requirements will be set up and defined in the section 3.

3 The multi-objective optimal control

3.1 Multi-objective optimal control design

The multi-objective optimal control (MOOP) design with the gain $\mathbf{K}_c = [k_1, \cdots, k_n]$ as design parameters was used for the system discussed in Section 2. The n is 4, 4 and 8 in the case (1), case (2) and case (3) respectively. The multi-objective optimal design problem is stated as,

$$\min\left\{\zeta_{max}, \mathbf{K}_c^T\mathbf{K}_c\right\} \tag{6}$$

Where, ζ_{max} denotes the maximum ratio of the real part to the modulus of eigenvalues of the matrix $\mathbf{A}_{cl}$ defined in Equation (5), and it describes the stability margin of linear systems. The term $\mathbf{K}_c^T\mathbf{K}_c$ denotes the *Frobenius* norm of the control gain matrix $\mathbf{K}_c$. Since the control system is optimized for zero initial conditions, the control effort cannot be included directly in the objective space and its *Frobenius* norm is used instead. By minimizing this norm, the total control energy is also minimized. To solve this MOOP, NSGA-II is used.

3.2 Multi-objective optimal control results

In order to optimize the motor active suspension parameters, the maximum damping ration ζ_{max} that can be achieved by MOOP and the control energy were chosen as optimization objectives. The suspension frequency f_{my} is used instead of the motor suspension stiffness. The calculation range of f_{my} is from 0 to 10 Hz and the range of motor suspension damping ratio ξ_{my} (positive sign indicates a stable value) is 0 to 1. Figure 2 shows the ζ_{max} of the bogie control system with different motor suspension parameters at the running speed of 350 km/h. Subgraph (a) shows that the optimal f_{my} of 3 Hz and ξ_{my} of 0.2 should be adopted in the bogie before applying active control. These paremeters can be understood by the mechanism of dynamic vibration absorber caused by the flexible suspension motor [15]. To optimize the motor suspension parameters in different control cases, the minimum ζ_{max} of the bogie control system with different motor suspension parameters was analyzed with the optimal control gains calculated by MOOP. That is, the achievable minimum ζ_{max} after applying control is the basis for selecting the motor suspension parameters. In the three control cases, a small f_{my} need be slected for a better control effect. In case (1), only the frame states are fed back and a large ξ_{my} is conducive to the system stability as shown in Subgraph (b). On the contrary, a small ξ_{my} should be adopted if only the motor states are fed back as shown in Subgraph (c). In these two cases, the maximum optimal f_{my} can reach 4 Hz. In case (3), the

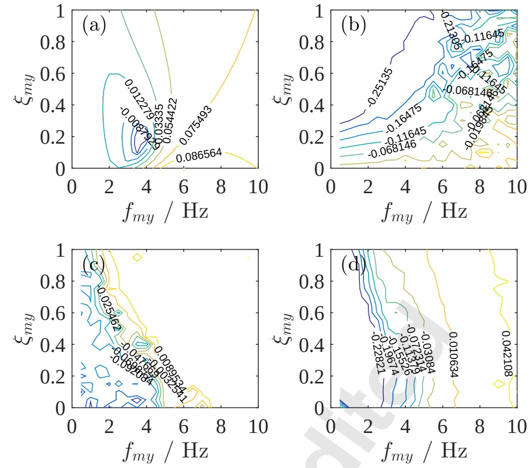

Fig. 2. The motor suspension parameter optimization in different cases. (a) Uncontrol; (b) Case (1); (c) Case (2); (d) Case (3).

states of the frame as well as the motor are fed back, the ξ_{my} has not so many limitations, but the maximum optimal f_{my} only can reach 2.5 Hz. According to the optimization results, the suggested f_{my} is set to 3 Hz, and ξ_{my} are set to 0.8, 0.2 and 0.3 in the three cases, separately.

The Pareto front of multi-objective optimization in different control cases are discussed here. Figure 3 shows the Pareto front of the bogie control system with the suggested motor suspension parameters. The Pareto front illustrates a conflicting relationship between ζ_{max} and $\mathbf{K}_c^T\mathbf{K}_c$. As the control effortincreases, the bogie is more stable. These results highlight the importance of the MOOP design. By having different design options, the decision-maker can choose any point from the solution set that meets the performance requirements. It can be seen from the figures that the linear system of bogie with fault of yaw damper is critical stable before applying the control at the speed of 350 km/h. The three control cases can effectively improve the system stability. The minimum ζ_{max} can be achieved up to -0.25, -0.10 and -0.23, respectively, after applying active control. The direct state feedback control can be seen as skyhook control, with which the corresponding vibration amplitude can be reduced. In case (1) and (3), suppressing the frame's vibration can significantly improve the running stability of bogie. In case (2), suppressing the motor's vibration is not very effective to improve the bogie running performance. However, the motor's displacement and velocity negative feedback are equivalent to increasing the motor lateral natural vibration frequency and damping ratio, separately. Therefore, in case (3), there is no need to choose so larger motor suspension damping as in case (1), and the optimal motor suspension frequency is reduced.

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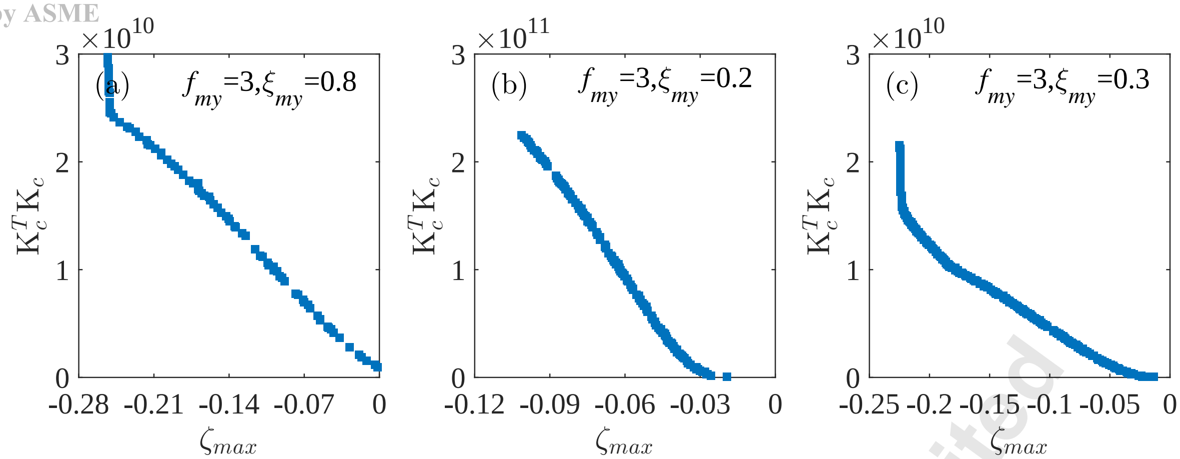

Fig. 3. The Pareto front in different control cases. (a) Case (1); (b) Case (2); (c) Case (3).

4 The linear stability analysis

4.1 Root locus analysis

The root locus of bogie linear system under frame lateral vibration control are analyzed to verify the effectiveness of active control in the entire speed range. In every control case, the feedback gains are got from calculated Pareto set based on a preset damping ratio ζ_{set}. In this study, the ζ_{set} of -0.15 in Pareto front is chosen. However, in some conditions, if the minimum ζ_{max} of the system with optimal gains calculated by MOOP is larger than -0.15, the maximum of the two values was chosen as ζ_{set} to get the feedback gains for the following analysis. The ζ_{set} can be stated as follows:

$$\zeta_{set} = \max\{\min(\zeta_{max}), -\mathbf{0.15}\} \tag{7}$$

Figure 4 shows the root locus of bogie linear system in different control cases. Each root locus is composed of 30 sets of characteristic roots in speed range of 20~600 km/h, and each "+" indicates a corresponding modal. The larger symbol represents a larger running speed. The horizontal axis indicates the modal damping ratio ζ (negative sign indicates a stable value), which is the ratio of the real part to the modulus of eigenvalues of the matrix $\mathbf{A}_{cl}$ defined in Equation (5). The vertical axis indicates the damped vibration frequency of modal, which is the imaginary part of eigenvalues. Normally, as the speed increases, the hunting-related modal damping reduces. When the ζ is greater than 0, the hunting movement is unstable. Mostly, the low frequency hunting modal determines the bogie hunting critical speed.

In the Figure, Subgraph (a) shows the root locus of the system with control case (1), in which the f_{my}, ξ_{my} are 3 Hz and 0.8 respectively. Subgraph (b) is a partial enlarged view of low frequency bogie hunting modal. It is clear that the hunting stability reduces with the increase of running speed, and the linear critical speed is 500 km/h.

Subgraph (c)~(f) show the system root locus in case (2) & (3), and the corresponding partial enlarged views of the bogie low frequency hunting modal. The bogie hunting stability improves in the two cases and the corresponding linear

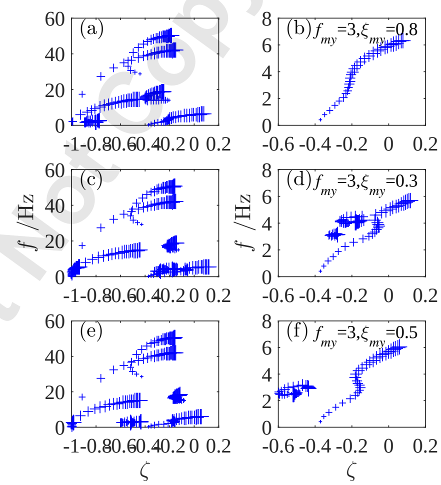

Fig. 4. Bogie root locus in different control cases. (a) Case (1); (b) Partial enlarged view of (a); (c) Case (2); (d) Partial enlarged view of (c); (e) Case (3); (f) Partial enlarged view of (e).

critical speeds are 430 and 500 km/h. Meanwhile, the shapes of the bogie hunting root locus curves in the three cases are significantly different. In case (1), as the speed increases, the bogie hunting damping ratio reduces monotonically. In case (2) and (3), the bogie hunting root locus curve likes the letter "s", and the stability reduces in some low-speed range. For the hunting stability, case (1) is better than case (3), and case (2) is the worst in the entire speed range.

4.2 The motor suspension parameter analysis

In order to compare the system performances in the three different feedback states, as well as the robustness of f_{my} and ξ_{my} against the perturbation of bogie system parameters, the influence of partial bogie dynamic parameters on the system stability in different cases are analyzed. In this study, the feedback gains are obtained from the Pareto set according to the 7. Figure 5 and Figure 6 show the calculated maxi-

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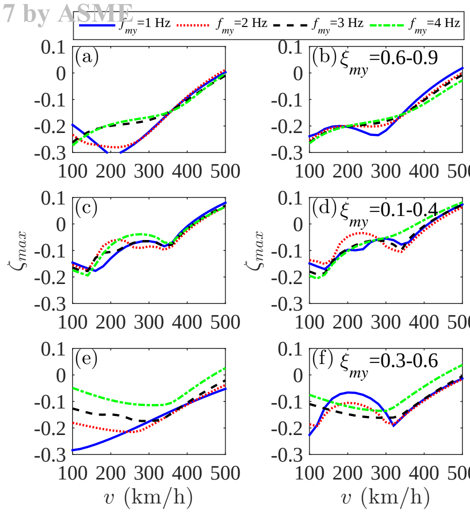

Fig. 5. The system stability with different motor suspension parameters and running speed; (a) Case (1), ξ_{my}=0.8; (b) Case (1), f_{my}=3 Hz; (c) Case (2), ξ_{my}=0.3; (d) Case (2), f_{my}=3 Hz; (e) Case (3), ξ_{my}=0.5; (f) Case (3), f_{my}=3 Hz.

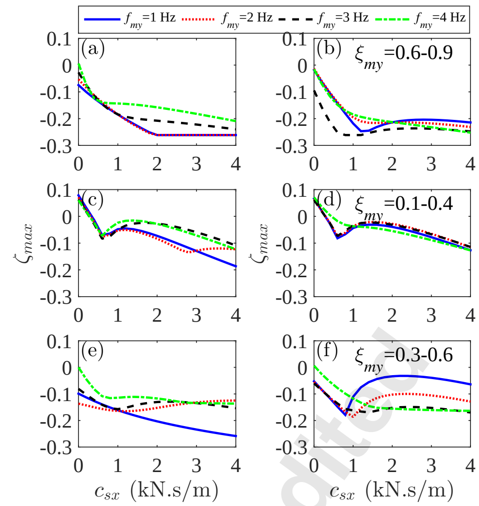

Fig. 6. The system stability with different motor suspension parameters and yaw dumping; (a) Case (1), ξ_{my}=0.8; (b) Case (1), f_{my}=3 Hz; (c) Case (2), ξ_{my}=0.3; (d) Case (2), f_{my}=3 Hz; (e) Case (3), ξ_{my}=0.5; (f) Case (3), f_{my}=3 Hz.

mum system damping ratio ζ_{max} of the bogie control system. In each subgraph, the horizontal axis is the varied dynamics parameter within a certain range, and the vertical axis is the ζ_{max} with different feedback states and motor suspension parameters.

Figure 5 reflects the influence of running speed v on the system stability. In case (1), a small f_{my} and a large ξ_{my} are conducive to the system stability. In case (2) and (3), the improper motor suspension parameters will reduce the system stability in the speed range of 200˜300 km/h. Especially in case (3), the f_{my} of 1˜2Hz and ξ_{my} of 0.5 should be adopted. Figure 6 shows the influence of yaw damper damping c_{sx} on the system stability. The stability improves with the increase of c_{sx} except for case (3) with a smaller ξ_{my}.

In short, the motor active flexible suspension with appropriate feedback states, gains and motor suspension parameters can improve the robustness of bogie hunting stability against the perturbation of system dynamics parameters, such as v, λ_e and c_{sx}, etc. The three control cases have different stability influence rules with the changing of system parameters. The robustness of control case (2) is the worst. The case (1) and (3) are more suitable for the dynamics performance requirements of vehicle. Meanwhile, a small f_{my} and appropriate ξ_{my} should be adopted to improve the system stability against the perturbation of bogie dynamics parameters.

5 The time-delay stability analysis

In a feedback control system, the time-delay caused by detecting and processing as well as response delay of actuator is inevitable. It is obvious that the effect of time-delay on the system dynamic performance. In this section, the influence of time-delay in the control system on bogie hunting stability is analyzed using the CTA method. In this study, the delayed part of the state vector is extended to 20 points, and the feedback gains in three cases are also obtained from the Pareto set according to the 7.

The root locus of the bogie control system considering the time-delay is analyzed. Figure 7 shows the root locus of the system including delay in feedback control. The CTA method is used to extend the dimension of the original system matrix. Compared to the system without delay, the time-delay system has more vibration modals with large damping ratio, but the large damping ratio does not affect the system dynamic performances. Additionally, the time-delay affects the damping ratio of low-frequency hunting modal, thus affecting the system stability. In the three cases, the delay makes the root locus curves of low frequency hunting modal at some low speeds shift to the left. Meanwhile, it also makes the root locus curves of low frequency hunting modals at high speeds shift to the right. So, the time-delay affects the system stability differently in different speed ranges. It can be understood that the time-delay reduces the system stability, but it also enhances the function of active dynamic vibration absorber of the lateral flexible suspension motor. So, the stability improves in a certain speed range, in which the bogie hunting frequency is close to the motor suspension frequency.

Figure 8 shows the maximum damping ratio ζ_{max} versus time-delay with the different motor suspension parameters. In the three cases, the system stability reduces significantly when the delay is larger than 10 ms. In some conditions, the delay slightly increases the system stability, but the motor suspension parameters have no very significant effects on the system stability caused by the time-delay in active control.

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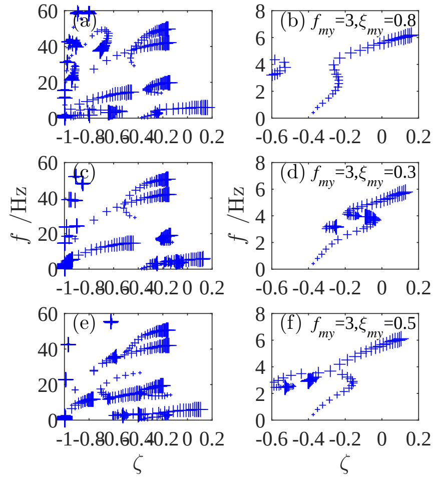

Fig. 7. Bogie root locus with time-delay of 15 ms in different control cases. (a) Case (1); (b) Partial enlarged view of (a); (c) Case (2); (d) Partial enlarged view of (c); (e) Case (3); (f) Partial enlarged view of (e).

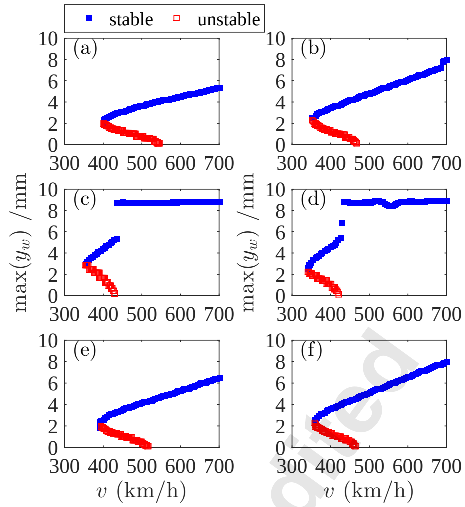

Fig. 9. The wheelset lateral displacement bifurcation curves about the running speed in different control conditions. (a) Case (1); (b) Case (1) with delay of 15 ms; (c) Case (2); (d) Case (2) with delay of 15 ms; (e) Case (3); (f) Case (3) with delay of 15 ms.

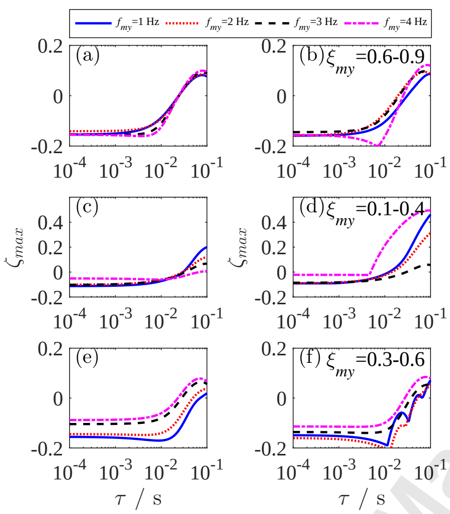

Fig. 8. The system stability with different motor suspension parameters and time delay; (a) Case (1), ξ_{my}=0.8; (b) Case (1), f_{my}=3 Hz; (c) Case (2), ξ_{my}=0.3; (d) Case (2), f_{my}=3 Hz; (e) Case (3), ξ_{my}=0.5; (f) Case (3), f_{my}=3 Hz.

6 The nonlinear stability analysis

The railway vehicle is a typical nonlinear dynamic system, which is mainly reflected in the nonlinear characteristics of the wheel-rail contact relationship and the suspension elements. The nonlinearity of wheel-rail contact includes the wheel-rail contact geometry and the saturation of wheel-rail creep force. The nonlinear critical speed is an important index for railway vehicles. The actual vehicle nonlinear critical speed depends on the external excitation. It is very important to analyze the nonlinear bifurcation behavior for understanding the vehicle lateral dynamic performances. In this section, a nonlinear bogie lateral dynamics model was established and the nonlinear bifurcation of wheelset lateral displacement versus running speed under the motor suspension active control was analyzed by a direct integral method.

The nonlinear characteristics of wheel-rail contact geometry and creep saturation are taken into account in the bogie nonlinear dynamics model. Based on the previous linear dynamics model, the wheel-rail contact equivalent conicity λ_e' can be expressed by a polynomial which is a nonlinear function about the wheelset lateral displacement y_w, as follows

$$\lambda_e' = a_0 + a_1 y_w^2 + a_2 y_w^6 + a_3 y_w^8 \tag{8}$$

Where, for a new wheel-rail contact, $a_0 = \lambda_e$, $a_1 = 2.0e3$, $a_2 = -1.0e9$, $a_3 = 5.0e15$ [16]. Shen's method was adopted to calculate the wheel-rail nonlinear creep force. In order to highlight the effects of active control on improving the bogie hunting stability, the fault of yaw damper was conducted.

The bogie hunting bifurcation can be obtained by using the direct integration of the nonlinear model with a changed initial value, which is an approximate periodic solution and was calculated by numerical integration. Figure 9 shows the bifurcation curves of wheelset lateral displacement versus running speed in different calculation conditions. The blue points in the figure represent the stable limit cycles, and the red points represent the unstable limit cycles. For the new wheel-rail contact, the bogie hunting has a typical subcritical bifurcation in all cases. The corresponding speed of a point where the vertical coordinate equals 0 is called the linear critical speed, and the minimum speed of the unstable limit cycle is the nonlinear critical speed.

With the fault of c_{sx}, the bogie nonlinear critical speed cannot meet the requirement of running at the speed of 350

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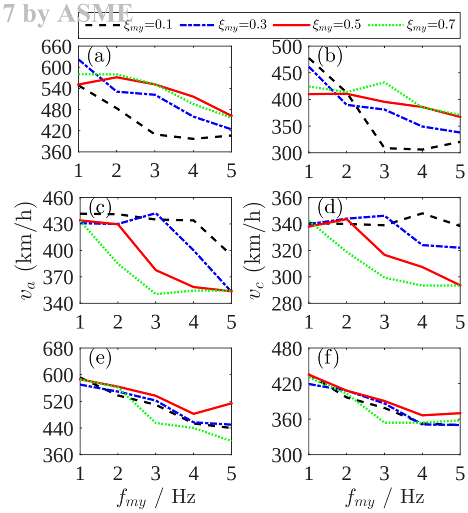

Fig. 11. The linear and nonlinear critical speed v_a, v_c with different motor suspension parameters in different cases. (a) Case (1), v_a; (b) Case (1), v_c; (c) Case (2), v_a; (d) Case (2), v_c; (e) Case (3), v_a; (f) Case (3), v_c.

km/h. In case (1) and (3), the feedback gains are obtained from the Pareto set according to the 7. If the time-delay is not taken into account, the bogie linear and nonlinear critical speeds are larger than 500 and 400 km/h respectively, which are equivalent to the stability of the uncontrolled bogie without fault of c_{sx}. When a delay of 15 ms is considered, the linear and nonlinear critical speeds reduced by about 60 and 40 km/h, respectively.

The preset ζ_{set} getting feedback gains has an important influence on the stability of bogie control system. The greater the ζ_{set} is used, the more stable the system is, but the greater control energy consumes. Figure 10 shows the relationship between the system critical velocity and ζ_{set} in different control cases. The linear and nonlinear critical velocity v_a, v_c are linear with the ζ_{set} basically.

Figure 11 shows the linear and nonlinear critical speeds with different motor suspension parameters. As the same as the previous analysis results, a small motor suspension stiffness is needed for a better system stability. The case (1) needs a larger suspension damping, but the case (2) needs a smaller suspension damping. In case (3), the motor suspension damping has no obvious effect on the system stability.

7 Conclusions

(1) In this study, the motor active flexible suspension was proposed to improve the hunting stability of a high-speed train bogie. The active effort acts between the frame and the motor based on three different feedback state configurations. In case (1) and (2), the frame's and motor's states are fed back respectively. For case (3), in which the frame's and motor's states are fed back together, there are eight measurements of motion, i.e. the lateral displacement and yaw angle and their first derivatives of frame, as well as the lateral displacements of two motors and their first derivatives. A partial state feedback control was applied to the underactuated and unobservable bogie system, and the multi-objective optimization with 2 objective functions about the stability index and the control effort was solved by the NSGA-II algorithm for getting the feedback gains.

(2) The linear stability of the bogie control system including a fault of yaw damper was analyzed. The three control cases can effectively improve the system stability, and the minimum ζ_{max} can be achieved up to -0.25, -0.10 and -0.23, respectively. The optimal motor suspension parameters in different cases are analyzed. In case (1), a large ξ_{my} is conducive to the system stability. On the contrary, a small ξ_{my} should be adopted in case (2). In these two cases, the maximum optimal f_{my} can reach 4 Hz. In case (3), there is almost no limit to the value of ξ_{my}, but the maximum optimal f_{my} only can reach 2.5 Hz. The direct state feedback control can reduce corresponding state's vibration amplitude. In case (1) and (3), suppressing the frame's vibration can significantly improve the running stability of bogie. In case (2), suppressing the motor's vibration is not very effective to improve the running performance. However, the motor's displacement and velocity negative feedback are equivalent to increasing the motor lateral natural vibration frequency and damping respectively. The time-delay in control system was considered, the system stability reduces significantly when the delay is larger than 10 ms.

(3) The bogie nonlinear hunting bifurcation was investigated by using the direct integration with the changed initial value. In case (1) and (3), the feedback gains are obtained from the Pareto set according to the preset ζ_{set} of 0.15. If the time-delay is not taken into account, the bogie linear and nonlinear critical speeds are larger than 500 and 400 km/h respectively, which are equivalent to the stability of the uncontrolled bogie without fault of c_{sx}. When a delay of 15 ms is considered, the linear and nonlinear critical speeds are reduced by about 60 and 40 km/h, respectively. The preset ζ_{set} getting feedback gains in MOOP has an important influence on the running stability and control energy. The linear and nonlinear critical velocity are linear with the ζ_{set} basically. Finally, the optimal motor suspension parameters in different control cases are verified by the nonlinear analysis.

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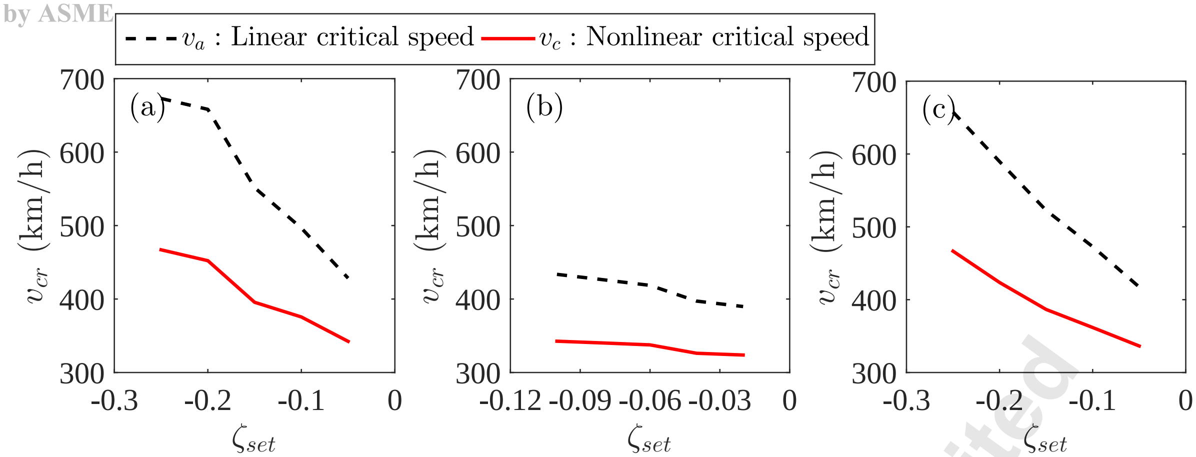

Fig. 10. The linear and nonlinear critical speed v_a, v_c with different preset ζ_{set} in different cases. (a) Case (1); (b) Case (2); (c) Case (3).

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